## Introduction le Stochastic Processes

 $(S_{J},F,P)$  (S,M)

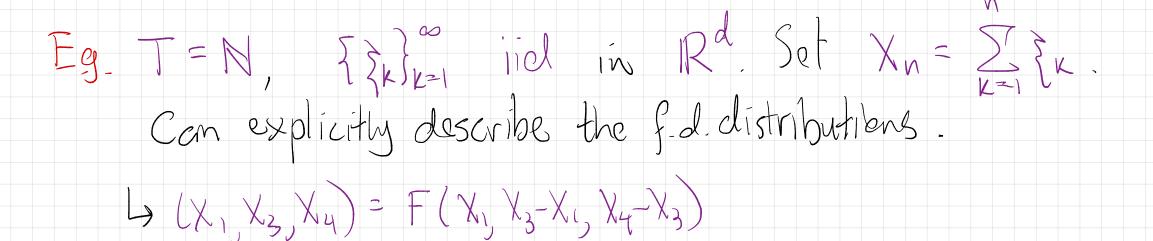
T & ordered set (with minimal element) T=N, T=[e,co), [a,b]

A stochastic process is a collection {Xtstet

of random variables X<sub>t</sub> = (S, F) > (SB).

The minimal information we'd like to have about a stochastic process is its finite-dimensional distributions:

For each finite set AST, the measures Lawp(XE)ter (Prob(S, B))



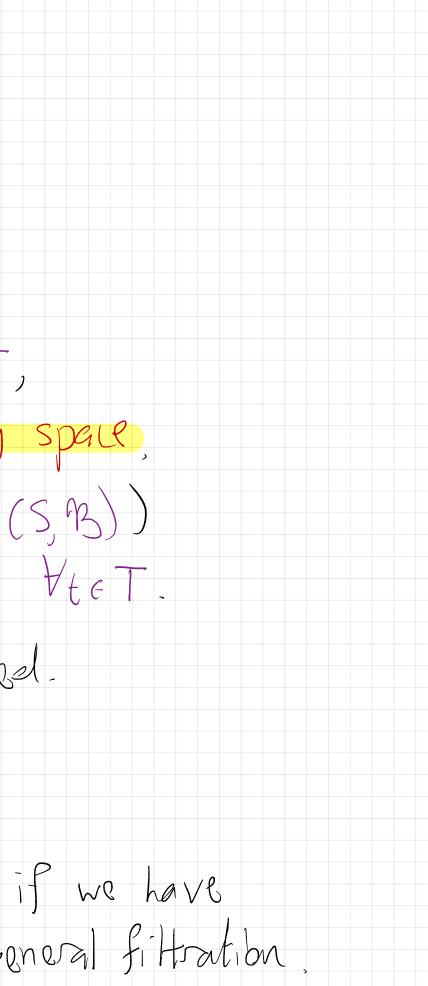
As we saw in [Lecture 34.2], it is often important

to understand 5(X1, X2, --, Xn) as n varies.

Def: A collection  $(F_t)_{t\in T}$  of  $\sigma$ -fields is called a filtration if  $F_s \subseteq F_t$  when set in T.

If  $(\Sigma, \mathcal{F}, \mathcal{P})$  is a probability space, and  $\mathcal{F}_t \leq \mathcal{F}$  viet, then  $(\Sigma, \mathcal{F}, (\mathcal{F}_t)_{t\in T}, \mathcal{P})$  is called a filtered probability space. A stochastic process  $(X_t)_{t\in T}$  (with  $X_t = (\Sigma, \mathcal{F}) \rightarrow (S, \mathcal{B})$ ) is called adapted if  $X_t$  is  $\mathcal{F}_t / \mathcal{B}$  - measurable  $\forall t \in T$ . Eq. If we set  $\mathcal{F}_t = \mathcal{T}(X_t)$  then  $X_t$  is clearly adapted.

Usually we can safely take this as the filtration. But if we have more than one process ground, we may need a more general filtration



Eq.  $\{\xi_{l}\}_{l=1}^{\infty}$  iid,  $X_{n} = \xi_{l} + \dots + \xi_{n}$ .  $F_{n} = \sigma(X_{l}, \dots, X_{n})$ .

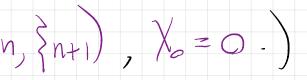
 $\mathbb{E}[g(X_n)|\mathcal{F}_k] = \mathbb{E}[g(X_k+\tilde{k}_{k+1}+\tilde{k}_n)|\sigma(X_{1,-1},X_k)]$ 

This process satisfies the Markov property. (Indeed, it fits the "random dynamics" model: Xn+1 = f(Xn, {n+1}), Xo=0.)

In the special case (on IRd) that the law of zi is

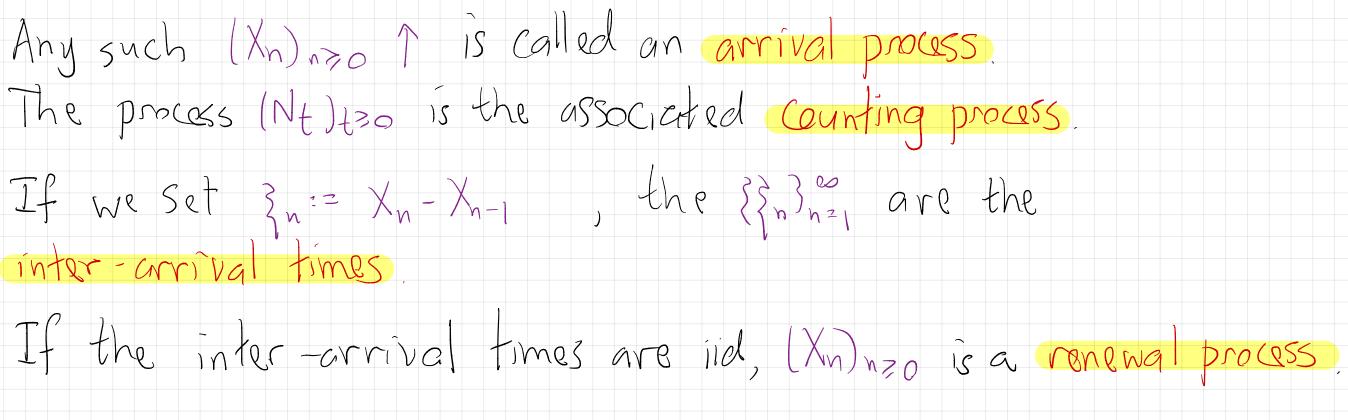
 $P(\frac{2}{1} = \pm e_j) = \frac{1}{2d} \quad |\leq j \leq d$ 

we call this stochastic process simple random walk.



F.g. Suppose (X), is a non-decreasing process

Define  $N_t = \sum_{n=1}^{\infty} I_{(3,t]}(X_n)$ 



The most important example of the counting process associated to) a renewal process is the Poisson process, which we'll study next time