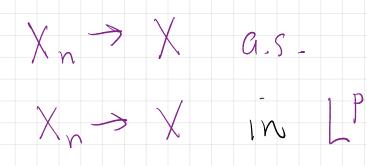
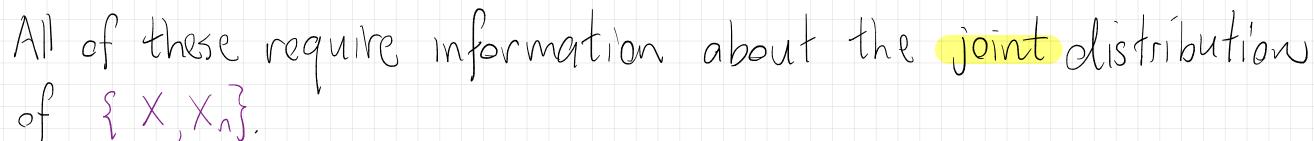
Convergence Revisited

We've considered several medes of Convergence

of random variables:







we're now going to turn to some onvergence notions that only care about the individual distributions.

Total Variation

Let {Mn3n=, be a sequence of probability measures on (S,B). They're just IR-valued functions on B, so we can use any function convergence notion we like.

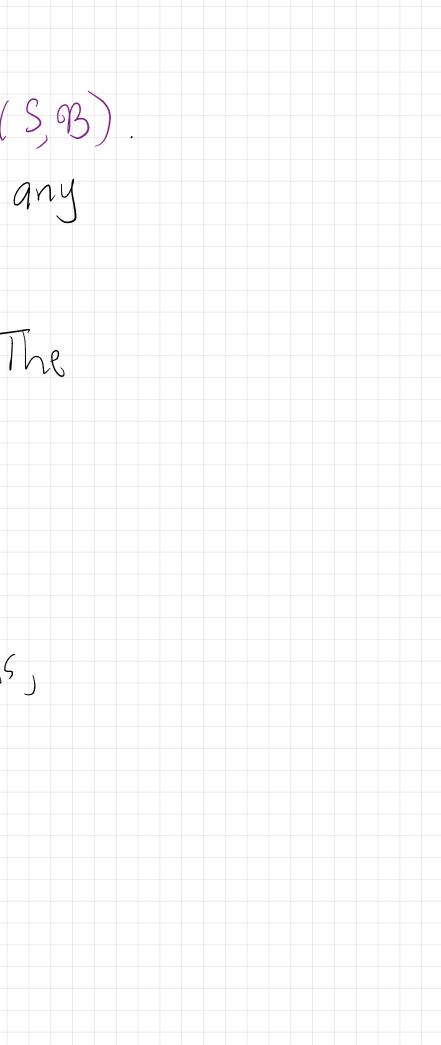
Def: Let u, v be probability measures on (S9B). The total variation distance between them is

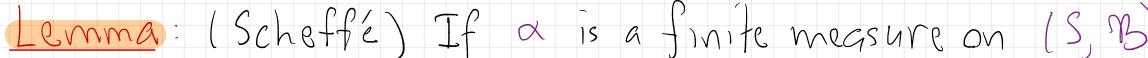
 $d_{TV}(M,V) = \sup_{B \in \mathcal{B}} |\mu(B) - \nu(B)|$

If XY are (S,B) - valued randem variables, we set

 $d_{TV}(X,Y) = d_{TV}(M_{X},M_{Y})$

= SUP | P(XEB) - P(YEB) | BEOB | P(XEB) - P(YEB) |





such that M, V << a with dy= uda, dv= vdo

 $d_{TV}(MV) = \frac{1}{2} || U - V ||_{L'(a)}$

Pf. For BeB,

 $|\mu(B) - \nu(B)|$

 $|\mu(B^{c}) - \nu(B^{c})|$

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Note: it is always possible to find such an d.

In fact: if {µn}n=1 is any countable Gliection of finite measures,

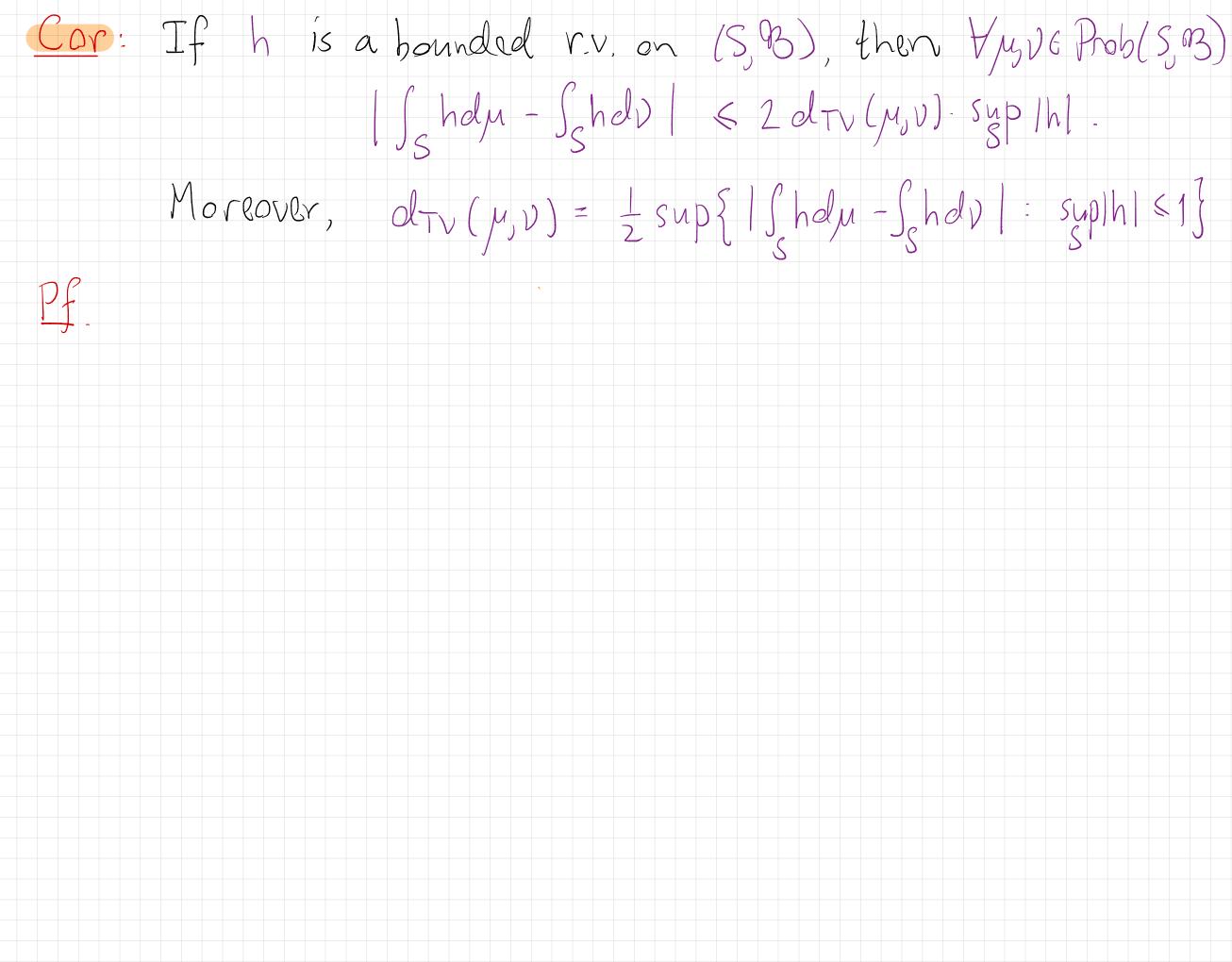
take $\alpha = \sum_{n=1}^{\infty} 2^{-n} M_n$.



- (This can be shown directly, but the present approach is slicker.) Let Syning C Prob(S, B). Fix a as above; then dyn= unda.
- · O= dTV (M1,M2) $\cdot d_{TV}(M_1, M_3)$

Pf.

· If Smilnz is dry - Cauchy, $d_{\mathrm{T}}(\mu_n,\mu_m) \rightarrow 0$



Total variation works well when S is countable.

Lemma: If S is countable, and M, VE Prob(S, B), then

 $d_{\mathrm{TV}}(\mu,\nu) = \frac{1}{2} \sum_{k \in S}^{T} |\mu(\{k\}) - \nu(\{k\})|$

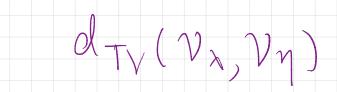
 $: \mathcal{M}_{n} \to \mathcal{T}_{V} \mathcal{M} \quad \text{iff} \quad \mathcal{M}_{n}(\{k\}) \to \mathcal{M}(\{k\}) \quad \forall k \in S.$



Eg. Mp^d Bernoulli(p).

dTV (Mp, Mg)

Eq. $\gamma_{\lambda} \stackrel{d}{=} Poisson(\lambda)$.



Eg. $d_{TV}(M_{P}, \mathcal{V}_{P})$