Math 280A: Fall 2020
Homework 8

Available	Monday, November 23	Due	Monday, November 30

Turn in the homework by 9:00pm on Gradescope. Late homework will not be accepted.

1. (Driver, Exercise 9.10) Suppose $(\Omega_j, \mathcal{F}_j)$, $j \in \{1,2\}$ are measurable spaces, and let $\pi_j \colon \Omega_1 \times \Omega_2 \to \Omega_j$ be the standard projections: $\pi_j(\omega_1, \omega_2) = \omega_j$.

- **(a)** Show that $\sigma\{\pi_1, \pi_2\} = \mathcal{F}_1 \otimes \mathcal{F}_2 := \sigma\{B_1 \times B_2 \colon B_1 \in \mathcal{F}_1, B_2 \in \mathcal{F}_2\}$. [To be clear: $\mathcal{B} := \sigma\{\pi_1, \pi_2\}$ is the smallest σ-field over $\Omega_1 \times \Omega_2$ with respect to which π_j is $\mathcal{B}/\mathcal{F}_j$-measurable for $j = 1, 2$.]
- **(b)** Let $\mathcal{E}_j \subseteq \mathcal{F}_j$ be subsets such that $\Omega_j \in \mathcal{E}_j$ and $\sigma(\mathcal{E}_j) = \mathcal{F}_j$ for $j = 1, 2$. Show that $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma\{A_1 \times A_2 \colon A_1 \in \mathcal{E}_1, A_2 \in \mathcal{E}_2\}$.

2. Let X be a real-valued random variable, with CDF F_X. Let $c > 0$. Prove that the function $t \mapsto F_X(t+c) - F_X(t)$ is in $L^1(\lambda)$, and that

$$\int_{\mathbb{R}} (F_X(t+c) - F_X(t))\,\lambda(dt) = c.$$

3. Let $\Omega = \{1,2,3,4\}$, $\mathcal{F} = 2^{\Omega}$, and $\mathbb{P}(A) = \frac{\#A}{4}$ for all $A \subseteq \Omega$. Give an example of two independent collections $\mathcal{C}_1, \mathcal{C}_2 \subset \mathcal{F}$ such that $\sigma(\mathcal{C}_1), \sigma(\mathcal{C}_2)$ are not independent.

4. Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of independent events. Show that

$$\mathbb{P}\left(\bigcap_{n=1}^{\infty} A_n\right) = \prod_{n=1}^{\infty} \mathbb{P}(A_n).$$