Math 280A: Fall 2020
Homework 1

Available	Monday, October 5	Due	Monday, October 12

Turn in the homework by 9:00pm on Gradescope. Late homework will not be accepted.

1. (Exercise 1.1.4 in Durrett) Suppose $\mathcal{F}_1, \mathcal{F}_2, \ldots$ are σ-fields over a sample space Ω, and suppose they are nested: $\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \ldots$ Prove that

$$\bigcup_{n=1}^{\infty} \mathcal{F}_j$$

is a field. Is it necessarily a σ-fields?

2. Let Ω be an infinite set. Let $\mathcal{F}$ denote the collection of all sets $E \subseteq \Omega$ such that either E or E^c is finite.

(a) Is $\mathcal{F}$ a field? Prove that your answer is correct.

(b) Is $\mathcal{F}$ a σ-field? Prove that your answer is correct.

3. (Exercise 4.8 in Driver) Let Ω be a set, and let $\mathcal{E}_1, \mathcal{E}_2 \subseteq 2^\Omega$ be collections of subsets of Ω. Show that $\sigma(\mathcal{E}_1) = \sigma(\mathcal{E}_2)$ if and only if $\mathcal{E}_1 \subseteq \sigma(\mathcal{E}_2)$ and $\mathcal{E}_2 \subseteq \sigma(\mathcal{E}_1)$. Give and example where $\sigma(\mathcal{E}_1) = \sigma(\mathcal{E}_2)$, but $\mathcal{E}_1 \neq \mathcal{E}_2$.

4. (Exercise 4.9 in Driver) Verify that the Borel σ-field $\mathcal{B}(\mathbb{R})$ is generated by any of the following collections of intervals:

(a) $\mathcal{E}_1 = \{(a, \infty) \colon a \in \mathbb{R}\}$

(b) $\mathcal{E}_2 = \{(a, \infty) \colon a \in \mathbb{Q}\}$

(c) $\mathcal{E}_3 = \{[a, \infty) \colon a \in \mathbb{Q}\}$

5. Let $(\Omega, \mathcal{F}, P)$ be a probability space, and suppose $A \in \mathcal{F}$ satisfies $P(A) > 0$. Define

$$\mathcal{F}_A := \{B \in \mathcal{F} \colon B \subseteq A\}, \text{and for } B \in \mathcal{F}_A \text{ define } P_A(B) = \frac{P(B)}{P(A)}.$$

Prove that $(A, \mathcal{F}_A, P_A)$ is a probability space.