MATH 280A: Probability Theory I

Thursday, Dec 10, 2020

* **Video Lectures:** 18.1, 18.2, 19.1, 19.2 posted on YouTube

* **Quizzes:** Regrade requests now active for all 5 quizzes. Will remain so until Dec 20 @ 8pm PST.

* **Final Exam:** Take-Home. Available: Sunday, Dec 13 @ 2:30pm PST.
  Due: Friday, Dec 18 @ 2:30pm PST.

* **Exit Survey:** Google form posted and emailed to you (shortly). Your feedback is appreciated!

$$\begin{tikzcd}
x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \cdots & x_n^{(1)} & \cdots \arrow[r] & x^1 \\
x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \cdots & x_n^{(2)} & \cdots \arrow[r] & x^2 \\
x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \cdots & x_n^{(3)} & \cdots \arrow[r] & x^3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & & \\
x_1^{(n)} \arrow[d] & x_2^{(n)} \arrow[d] & x_3^{(n)} \arrow[d] & \cdots & x_n^{(n)} \arrow[d] & \cdots \arrow[r] & x^n \\
x_1 & x_2 & x_3 & & x_n & &
\end{tikzcd}$$

$(x^m)_{m\geq 1}^{\infty}$ sequence of "points" in Q

$x_1^{m_j(k)} \dots$ $\therefore$ $k \to m_1(k)$ $\uparrow \infty$ s.t. $x_1^{m_1(k)} \to x_1 \in [0,1]$. $x_1^{m_2(k)} \to x_1$

$x_2^{m_j(k)} \dots$ $k \to m_2(k)$ $\uparrow \infty$ $\{m_2(k) : k \in \mathbb{N}\} \subseteq \{m_1(k) : k \in \mathbb{N}\}$

s.t. $x_2^{m_2(k)} \to x_2$

$x_{j-2}^{m_j(k)} \to x_{j-2}$

$x_{j-1}^{m_j(k)} \to x_{j-1}$

$\lim_{k\to\infty} x_j^{m_j(k)}$

$x_j^{m_j(k)} \xrightarrow{k\to\infty} x_j$ for each j.

$|x_j^{m_j(j)} - x_j|$

$\leq |x_j^{m_j(j)} - x_j^{m_j(k)}| + |x_j^{m_j(k)} - x_j|$

$\vdots$

$x_k^{m_k(k)} \to x_k$ $\forall k$.

$\{m_j(j) : j = 1 \dots \infty\}$

$\cup\!\!\mid$

$\{m_j(k) : k \geq j\}$

$\varepsilon/2$

Find K large enough that $< \varepsilon/2$ $\forall k \geq K$.

$|x_1^{m_1(1)} - x_1|$

$d\mu_n = f_n \, d\lambda$

$$\mu_1 \otimes \cdots \otimes \mu_d \, (B) \qquad B \in \mathcal{B}(\mathbb{R}^d)$$

$$B = B_1 \times \cdots \times B_d \qquad B_j \in \mathcal{B}(\mathbb{R})$$

$$= \mu_1(B_1)\,\mu_2(B_2) \cdots \mu_d(B_d)$$

$$= \int_{B_1} f_1 \, d\lambda \cdot \int_{B_2} f_2 \, d\lambda \cdots \int_{B_d} f_d \, d\lambda$$

$$= \int_{B_1 \times \cdots \times B_n} f_1 \otimes f_2 \otimes \cdots \otimes f_d \; d(\lambda \otimes \cdots \otimes \lambda)$$

Prob. density on $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d))$

$$\mu_1 \otimes \cdots \otimes \mu_d \;\; \text{on } B_1 \times \cdots \times B_d, \quad B_j \in \mathcal{B}(\mathbb{R}^d)$$

$$\shortparallel$$

$$d\nu = f_1 \otimes \cdots \otimes f_d \, d\lambda^d$$

π-system.

?; agree on $\sigma\{B_1 \times \cdots \times B_d\}$

$=?\ \mathcal{B}(\mathbb{R}^d)$

$$\mu_1 \otimes \cdots \otimes \mu_d (A) \qquad A \in \mathcal{B}(\mathbb{R})$$

$$= \int \mathbb{1}_A \, d(\mu_1 \otimes \cdots \otimes \mu_d)$$

$$= \int d\mu_d \int d\mu_{d-1} \cdots \int d\mu_1 \, \mathbb{1}_A$$

$$= \int \lambda(dx_d) f_d(x_d) \cdots \int \lambda(dx_2)\, f_2(x_2) \int f_1(x_1)\, \mathbb{1}_A(x_1, \dots, x_d)\, \lambda(dx_1)$$

$$= \int \lambda(dx_1) \int \lambda(dx_2) \cdots \int \lambda(dx_d)\, \overbrace{f_1(x_1) f_2(x_2) \cdots f_d(x_d)\, \mathbb{1}_A(x_1, \dots, x_d)}^{f_1 \otimes \cdots \otimes f_d \, \mathbb{1}_A}$$

$$= \int_A f_1 \otimes \cdots \otimes f_d \; d(\lambda \otimes \cdots \otimes \lambda)$$