MATH 280A: Probability Theory I

Tuesday, Nov 3, 2020

* **New Meeting Time:** 1 – 1:50 pm Tuesdays & Thursdays (except quiz days)

* **TA Office Hours:** Extended (2 hrs → 4 hrs)

* **Video Lectures:** 9.1, 9.2 posted on YouTube

* **Quiz 3:** Thursday, Nov 5, 1-1:50pm or 7-7:50pm (Google Form)
  Covering up to Lecture 8.2.

* **HW5:** Due Monday, Nov 9, by 9pm.

HW4
#2

$$\sigma(X_1,\dots,X_d) = \underline{X}^* \mathcal{B}(\mathbb{R}^d) \qquad \underline{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_d \end{bmatrix}$$

$\|$

smallest σ-field wrt which $X_1,\dots,X_d$ is measurable.

Contains $X_j^{-1}((a,b])$ $-\infty \le a \le b \le \infty$

$$\underline{X}^* \mathcal{B}(\mathbb{R}^d) = \{ \underline{X}^{-1}(E) : E \in \mathcal{B}(\mathbb{R}^d) \}$$

Consider $E = \underbrace{(a_1,b_1] \times (a_2,b_2] \times \cdots (a_d,b_d]}_{\mathcal{R}^d}$

$$\underline{X}^* \mathcal{B}(\mathbb{R}^d) = \underline{X}^* (\sigma(\mathcal{R}^d))$$
$$= \sigma(\underline{X}^*(\mathcal{R}^d))$$

$\underline{X}^* \mathcal{R}^d = \{ \underline{X}^{-1}(\quad) \}$

$= X_1^{-1}(a_1,b_1] \cap X_2^{-1}(a_2,b_2] \cap \cdots \cap X_d^{-1}(a_d,b_d]$

take $a_2 = \cdots = a_d = -\infty$
$b_2 = \cdots = b_d = \infty$.

$X_2^{-1}(a_2,b_2]$
$X_1^{-1}(a_1,b_1] \rightarrow$

HW3
#2

Prop: $\nexists$ function $m: 2^{\mathbb{R}} \to [0,\infty)$

s.t. (1) $m\left(\bigsqcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} m(A_j)$

(2) $m(\phi) = 0$, $m([0,1]) = 1$

(3) $m(E+x) = m(E)$ $\forall E \in 2^{\mathbb{R}}$ $\forall x \in \mathbb{R}$,

$m = \lambda^*$ satisfies (2,3) $\therefore$ $\cancel{(1)}$

$\mathbb{R}/\mathbb{Q}$ $\quad x \sim_{\mathbb{Q}} y$ iff $x - y \in \mathbb{Q}$

Chose exactly 1 rep from each equiv class in $\mathbb{R}/\mathbb{Q}$ $\rightsquigarrow$ form Φ.

Show that if (1), $m(\Phi) = 0$ and ∞.

Q: What is $\lambda^*(\Phi)$? Undetermined.

Hw3

#1 :

(X, d) pseudo-metric space

$\emptyset \neq V \subseteq X$ $\quad f : V \to \mathbb{R}$ Lip-K.

$\forall x, y \in V \quad |f(x) - f(y)| \leq K d(x,y)$

$\overline{V} = \{x \in X : \exists (v_n) \subseteq V$ with $v_n \xrightarrow{d} x\}$

$f(\lim_{n\to\infty} v_n) = \bar{f}(x) := \lim_{n\to\infty} f(v_n)$

(1) well-defined $\left\{ \begin{array}{l} \text{limit exists} \checkmark \\ \text{this gives a good definition} \checkmark \end{array} \right.$

(2) $\bar{f}$ is Lip-K.

1.1 $|f(v_n) - f(v_m)| \leq K d(v_n, v_m) \to 0$. $\therefore (f(v_n))_{n\geq 1}^{\infty}$ is Cauchy.

$\therefore$ limit exists.

$v_n \to x \therefore d(v_n, v_m) \leq d(v_n, x) + d(x, v_m) \to 0 + 0$.

1.2. If $\left. \begin{array}{l} v_n \to x \\ w_n \to x \end{array} \right\} |f(v_n) - f(w_n)| \to 0$

$|f(w_n) - f(v_n)| \to 0$.

$\leq K d(v_n, w_n)$

$d(v_n, w_n) \leq d(v_n, x) + d(x, w_n)$

$\to 0$.

$\checkmark$

Let $x, y \in \overline{V}$

Choose $v_n \to x$

$w_n \to y$

$$|\bar{f}(x) - \bar{f}(y)|$$

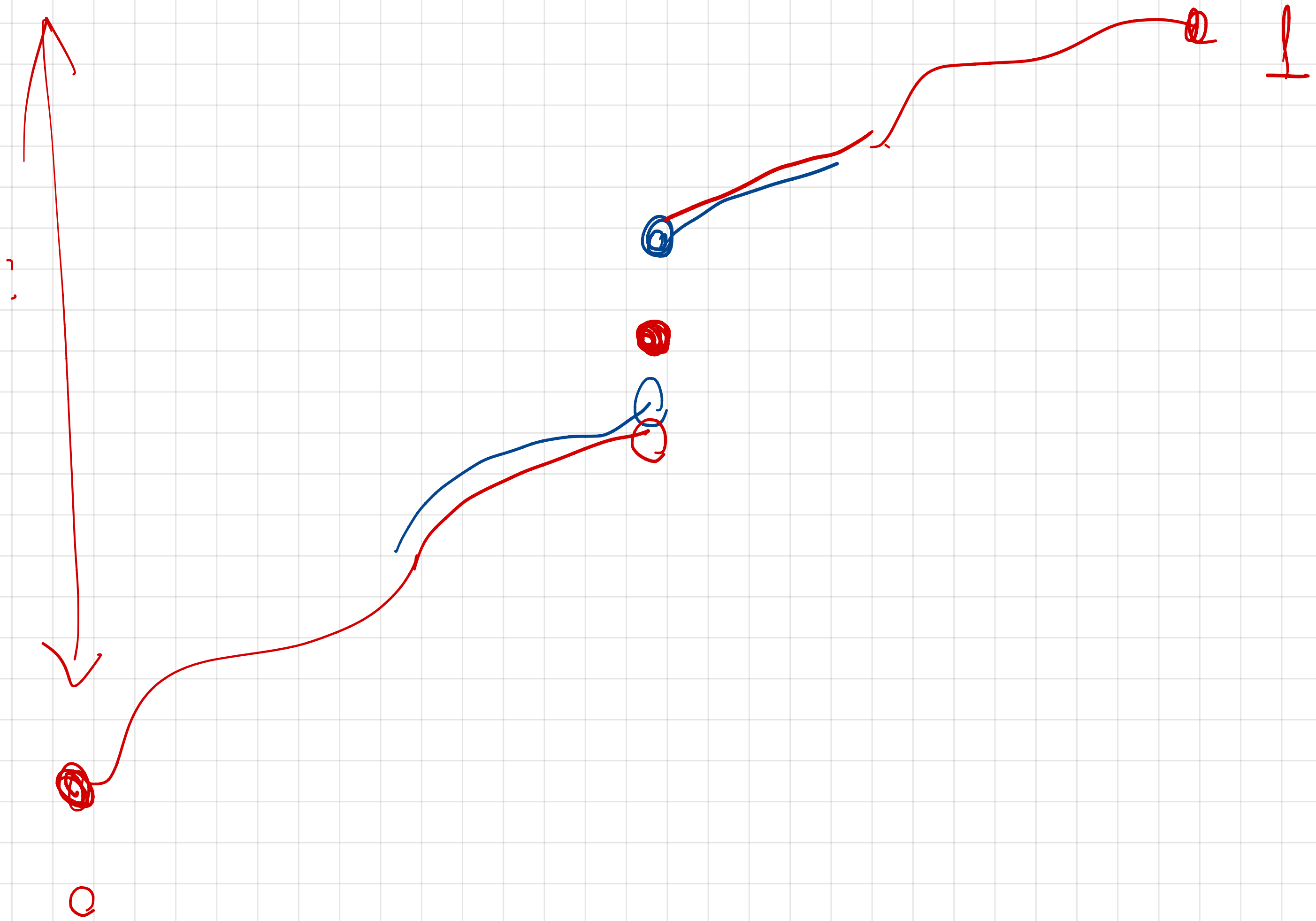
$$= \left|\lim_{n\to\infty} f(v_n) - \lim_{n\to\infty} f(w_n)\right|$$

$$= \lim_{n\to\infty} \underbrace{|f(v_n) - f(w_n)|}_{\leq K d(v_n, w_n)}$$

$$\leq K \underbrace{\lim_{n\to\infty} d(v_n, w_n)}_{= d(x,y)}$$

HW4

#5:



$$M = F(1) - F(0) = G(1) - G(0)$$

$x_1, x_2, x_3, \ldots$ discont. pts.

$$\sum_{j=1}^{\infty} F(x_j+) - F(x_j-) \le M.$$

$$\begin{cases} h_j^1 = F(x_j+) - F(j-) \\ h_j^2 = G(x_j+) - G(j-) \end{cases}$$

$$h_j^k \to 0, \quad j \to \infty.$$

$$\sum_{j=1}^{n} h_j < \infty \qquad \sum_{j=k}^{\infty} h_j \to 0$$

as $k \to \infty$.