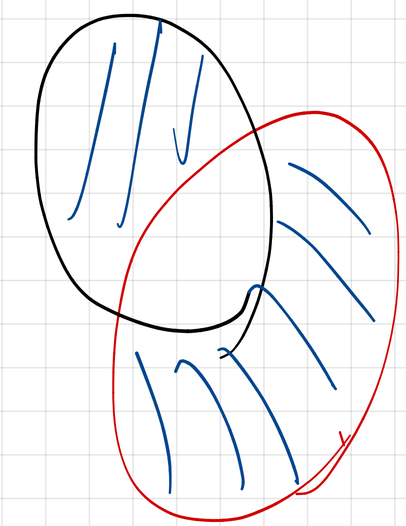
MATH 280A: Probability Theory I

Tuesday, October 6, 2020

* **Video Lectures:** 0, 1.1, 1.2 posted on YouTube
* **Quiz 1:** Thursday, October 8, 1–1:50pm or 7–7:50pm
  - Sign up for your quiz time: Google form
  - The quiz will be posted on Gradescope; you will turn it in on Gradescope.
* **HW 1:** Due Monday, October 12, 9pm.

Question: S'pose $\mathcal{A} \subseteq 2^{\Omega}$ satisfies:



(1) $\Omega \in \mathcal{A}$

(2) $\mathcal{A}$ is closed under Δ

$$A \Delta B = A \setminus B \cup B \setminus A$$
$$(A \cap B^c) \cup (B \cap A^c)$$

(3) $\mathcal{A}$ is closed under $\cap$

Is $\mathcal{A}$ a field? Yes. $\Omega \setminus A$

Complement: is $A^c = A \Delta ?$

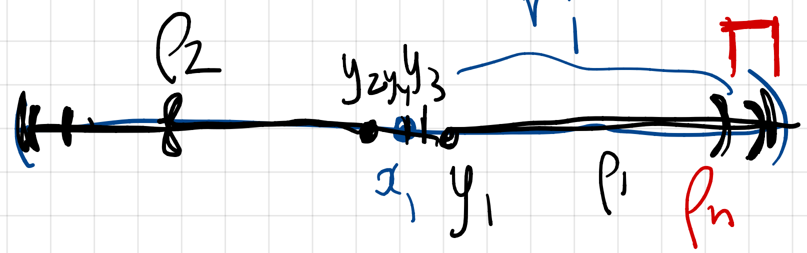
$$\Omega \Delta A = \underbrace{(\Omega \cap A^c)}_{A^c} \cup \underbrace{(A \cap \Omega^c)}_{\emptyset} = A^c$$

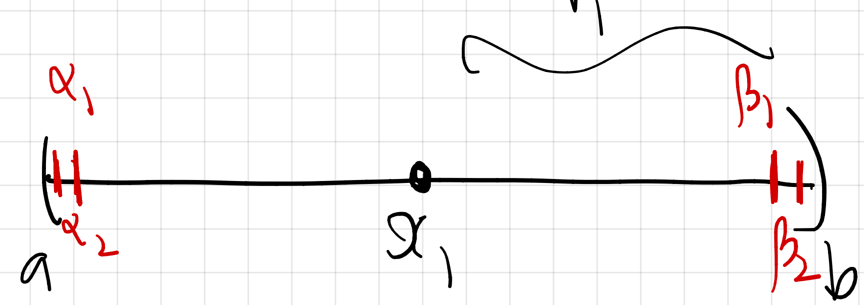
Open set $U \subseteq \mathbb{R}^d$

$$U = \bigcup_{j=1}^{\infty} B(x_j, r_j)$$

$\mathbb{Q}$

$$B(x_1, r_1) = \bigcup_{i=1}^{\infty} B(y_i, \rho_i)$$





$\alpha_j, \beta_j \in \mathbb{Q}$

$a < \alpha_j < \beta_j < b$

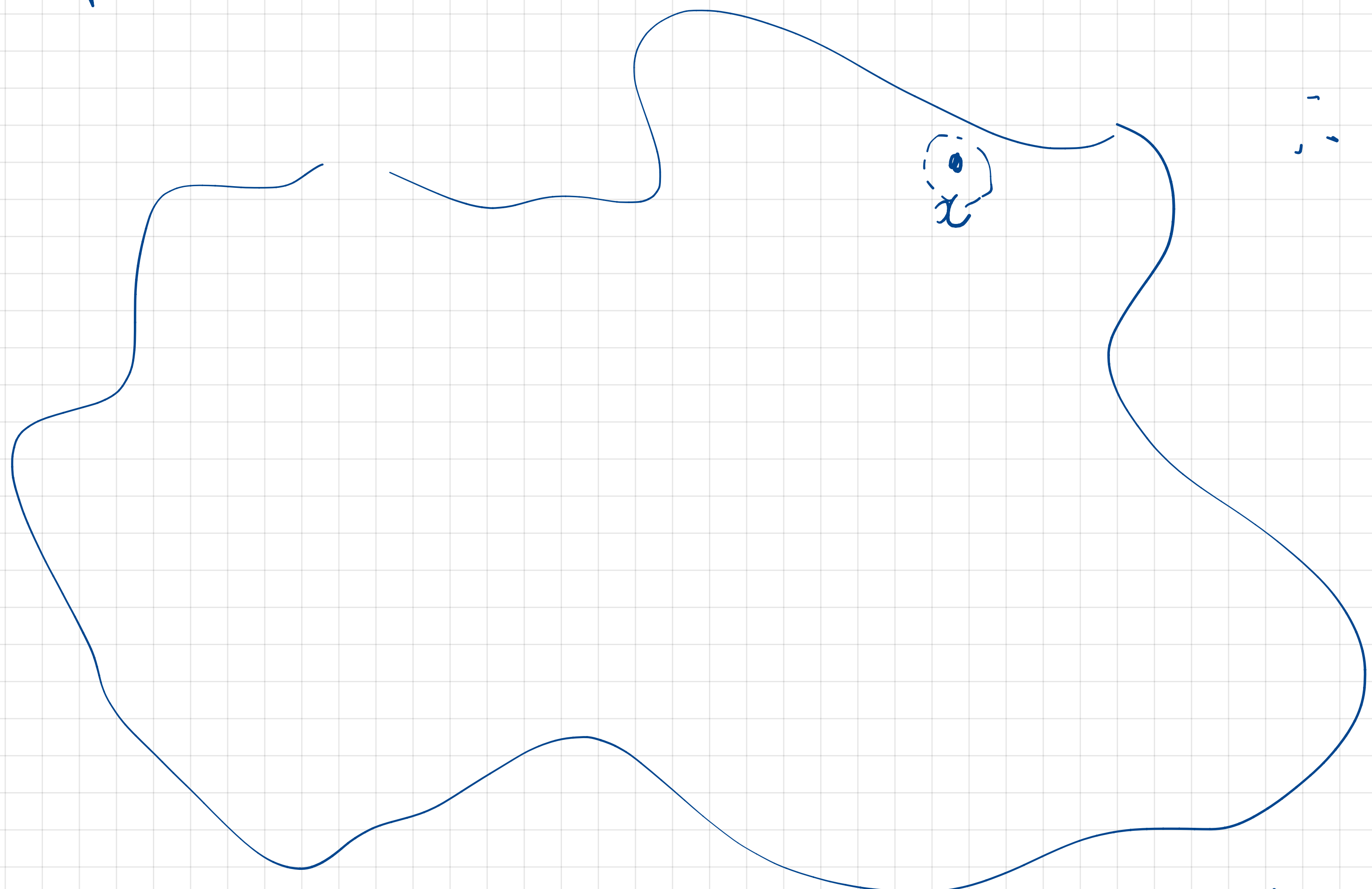
$|a - \alpha_j|, |\beta_j - b| < \frac{1}{j}$

$$B(y_j, \rho_j) = B\left(\frac{\alpha_j + \beta_j}{2}, \frac{\beta_j - \alpha_j}{2}\right) = (\alpha_j, \beta_j)$$

$U \subseteq \mathbb{R}^d$

open

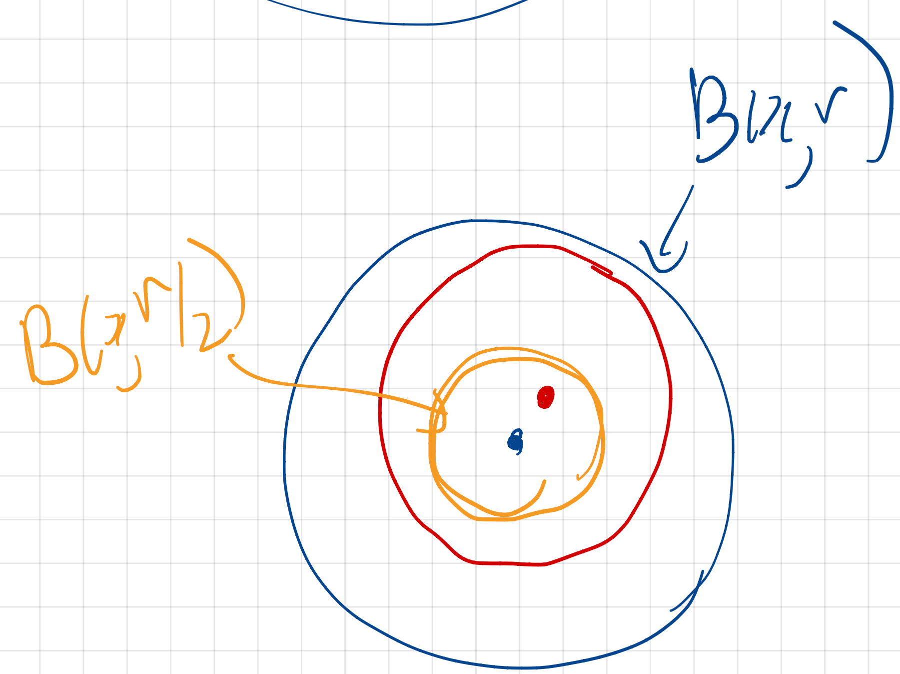
B/c U open, $\exists r_x > 0$ s.t. $\underbrace{B(x, r_x)}_{B_{in}} \subseteq U$.



$\therefore U = \bigcup_{x \in U} B(x, r_x)$

replace $B(x, r_x)$ by $B(y, r)$

rational



Claim:

$U = \bigcup_{x \in U}$ B assoc. to x

$\to \quad \mathcal{S} = \bigcap \{ \mathcal{F} : \mathcal{F} \text{ is a florg} \}$

[the set of florgs is closed under arbitrary $\bigcap$]

is the "smallest florg"

(1) $\mathcal{S}$ is a florg

(2) If $\mathcal{F}_0$ is any florg, then $\mathcal{S} \subseteq \mathcal{F}_0$.

[florg = σ-field over Ω, containing $\mathcal{E}$]

$\mathcal{S} = \bigcap \{\text{all florgs}\}$

$\mathcal{F}_0$ is in this list.