MATH 280A: Probability Theory I

Tuesday, October 27, 2020

* **New Meeting Time:** 1 – 1:50 pm Tuesdays & Thursdays (except quiz days)

* **Updated Course Policies:** Lecture dates, HW regrades – see webpage

* **Video Lectures:** 7.1, 7.2, 8.1, 8.2 posted on YouTube

* **HW4:** Due Monday, Nov 2, by 9pm.

Y is $\sigma(X_1, \dots, X_d) / \mathcal{B}(\mathbb{R})$ - meas.

Define $f \equiv \limsup_{n\to\infty} f_n$.

$\infty > Y = \lim_{n\to\infty} Y_n = \limsup_{n\to\infty} f_n(\underline{X}) \longleftarrow f(\underline{X}) < \infty$

simple meas. $\therefore Y_n = f_n(\underline{X})$ $\qquad$ i.e. $f < \infty$ on $\underline{X}(\Omega)$.

want $f(\mathbb{R}^d) < \infty$.

Actually define $f(\underline{t}) = \begin{cases} \limsup_{n\to\infty} f_n(\underline{t}) & \text{if } \underline{t} \in \underline{X}(\Omega) \\ 0 & \text{otherwise.} \end{cases}$

$\underline{t} \in \mathbb{R}^d$

Can have $f, g : \mathbb{R}^d \to \mathbb{R}$, $f \neq g$ s.t. $f(\underline{X}) = g(\underline{X})$

HW3, 3(a) $\quad \gamma \cdot \mathcal{B} \overset{?}{\subseteq} \mathcal{B}(\mathbb{R})$

$$\overset{\cap}{\mathcal{B}(\mathbb{R})}$$

$f:(\Omega, \mathcal{F}) \to (S, \mathcal{B})$

$$f^*\mathcal{B} = \{ f^{-1}(B) : B \in \mathcal{B} \}$$

$\Omega = S = \mathbb{R}$, $f(x) = x/\gamma$.

$\mathcal{E} = \mathcal{B}_{(]}$.

If $\mathcal{B} = \sigma(\mathcal{E})$ then

$\Leftarrow \quad f^*(\sigma(\mathcal{E})) = \sigma(f^*\mathcal{E})$

$$f^*(\mathcal{B}(\mathbb{R})) = \sigma(f^*\mathcal{B}_{(]})$$

$$= \sigma\{ f^{-1}(A) : A \in \mathcal{B}_{(]} \} = \sigma\{ A' : A' \in \mathcal{B}_{(]} \}$$

$$\gamma \cdot A$$

$$\| $$

$$\{f^{-1}(B) : B \in \mathcal{B}(\mathbb{R})\}$$

$$\|$$

$$\gamma \cdot \mathcal{B}$$

$$\left[A = \bigcup_j (a_j, b_j] \right] = \mathcal{B}(\mathbb{R})$$

$$A' = \gamma \cdot A = \bigcup_j (\gamma a_j, \gamma b_j] \in \mathcal{B}_{(]}.$$

$(\gamma > 0)$