MATH 280A: Probability Theory I

Tuesday, October 20, 2020

* **Video Lectures:** 5.1, 5.2, 6.1, 6.2 posted on YouTube

* **Quiz 2:** Thursday, 1-1:50pm or 7-7:50pm. [Google Form]
  ↳ Covering up to Lecture 5.2.

* **HW1:** Grades published.
  ↳ Regrade window:

Wednesday, 10/21 8am
→ Friday, 10/23 8pm

* **HW3:** Due Monday, October 26, 9pm.

$C = \bigcap_{n=1}^{\infty} C_n$

is closed.

$\in \mathcal{B}(\mathbb{R})$.

"Most" subsets of C are not $\in \mathcal{B}(\mathbb{R})$

$\# C = \# \mathbb{R} = 2^{\aleph_0}$

$\therefore \# 2^{C} = 2^{\# C} = 2^{2^{\aleph_0}} >>>> \# \mathcal{B}(\mathbb{R}) = 2^{\aleph_0}$

$(\mathbb{R}, \mathcal{B}(\mathbb{R}), \lambda) \xrightarrow{\text{extend}} (\mathbb{R}, \mathcal{F}, \bar{\lambda})$ or $\mathcal{F} = \overline{\mathcal{B}([0,1])}^{d_\mu}$ ← is complete in terms of nullsets.

λ can't see most subsets of C.

$\lambda(C) = 0$.

HW 2 Problem 5

$(\Omega, \sigma(\mathcal{A}), \mathbb{P})$ Find $A \in \mathcal{A}$ s.t. $\mathbb{P}(A \triangle B) < \varepsilon$.

$\Downarrow$

B

Show that $\mathcal{C} := \{B \in \sigma(\mathcal{A}) : \forall \varepsilon > 0 \ \exists A \in \mathcal{A} \text{ with } \mathbb{P}(A \triangle B) < \varepsilon\} \supseteq \mathcal{A}$

is a σ-field.

- $\mathbb{P}(\emptyset \triangle \emptyset) = 0 < \varepsilon$ ✓ $\therefore \emptyset \in \mathcal{C}$ ✓

  $A = \emptyset$

- $\mathbb{P}(A^c \triangle B^c)$

  $A^c \triangle B^c = (A^c \cap (B^c)^c) \sqcup (B^c \cap (A^c)^c)$

  $= (A^c \cap B) \sqcup (B^c \cap A) = B \triangle A = A \triangle B$

  $\|$

Find $\underline{\underline{A}}$ s.t. $\mathbb{P}(A \triangle B) < \varepsilon$ $\quad \therefore B^c \in \mathcal{C}$. ✓

$\cap$

$\mathcal{A}$

$\mathcal{C}$ ✓ $\quad \sum_{n=1}^{\infty} \mathbb{P}(A_n \triangle B_n) < \varepsilon$.

$\Downarrow$

$\mathbb{P}(A \triangle B) = \mathbb{P}\left(\left(\bigcup_n^N A_n\right) \triangle \left(\bigcup_n B_n\right)\right)$

- $(B_n)_{n=1}^{\infty}$ in $\mathcal{C}$; $\boxed{B = \bigcup_{n=1}^{\infty} B_n}$

  Find $A_n \in \mathcal{A}$ s.t. $\mathbb{P}(A_n \triangle B_n) < \frac{\varepsilon}{2^n}$ $\quad \therefore A = \bigcup_n A_n$