MATH 280A: Probability Theory I

Tuesday, October 15, 2020

* **Video Lectures:** 4.1, 4.2, 4.3 posted on YouTube

* **HW2:** Due Monday, October 19, 9pm.
  - ↳ Former Problem 5 removed; reserved to HW3.
  - ↳ New Problem 3 added.

* **Zoom Hour:** Didn't happen last night (internet problems)
  - ↳ Try tonight: Thursday, 10/15, 8:30-9:30pm PT.

$A_n \in \mathcal{A}$, $A_n \uparrow A \in \mathcal{A}_\sigma$

$\mu(A_n) \longrightarrow \mu^*(A)$, and $\mu^*(A \setminus A_n) \to 0$.

False if μ is not finite.

If X is a nonempty set in $\mathbb{R}$, $\alpha = \inf X$,

then for any $z > \alpha$, , there must exist $x \in X$

s.t. $\alpha \le x \le z$.

$\mu^*(E)$

For any $\varepsilon > 0$, $\exists\, x \in X$ s.t. $x \le \alpha + \varepsilon$.

(Otherwise, $\alpha + \varepsilon$ would be a lower bound for X.

$\alpha = \inf X \ge \alpha + \varepsilon$ $\lightning$

glb X

$(\Omega, 2^{\Omega}, \mu)$

$\mu^* \rightsquigarrow d_\mu$

$[2^{\Omega}]_{\sim}$ d_μ becomes a metric.

$E_1, E_2 \subseteq \Omega$ $E_1 \sim E_2$ iff $d_\mu(E_1, E_2) = 0$.

If $E_1, E_2 \in \cancel{\sigma(\mathcal{A})}$ $\bar{\mathcal{A}}$ $\bar{\mu}(E_1 \Delta E_2) = \mu^*(E_1 \Delta E_2)$

$\overline{\mathcal{A} \ni A_n \to A \neq B}$

$A_n \to B$

$\mu : \mathcal{A} \to \mathbb{R}$

$\bar{\mu} : \bar{\mathcal{A}} \to \mathbb{R}$

for any $\{A_n\}$ s.t. $A_n \to A$.

$\bar{\mu}(A) := \lim_{n \to \infty} \mu(A_n)$