

# MATH 280A: Probability Theory I

Tuesday, October 15, 2020

\* **Video Lectures**: 4.1, 4.2, 4.3 posted on YouTube

\* **HW2**: Due Monday, October 19, 9pm.

↳ Former Problem 5 removed; reserved to HW3.

↳ New Problem 3 added.

\* **Zoom Hour**: Didn't happen last night (internet problems)  
↳ Try tonight: Thursday, 10/15, 8:30-9:30pm PT.

$A_n \in \mathcal{A}$ ,  $A_n \uparrow A \in \mathcal{A}_\sigma$

$\mu(A_n) \rightarrow \mu^*(A)$ , and

$\mu^*(A \setminus A_n) \rightarrow 0$ .

false if  $\mu$  is not finite.

If  $X$  is a nonempty set in  $\mathbb{R}$ ,  $\alpha = \inf X$ ,  
then for any  $\varepsilon > 0$ , there must exist  $x \in X$   
s.t.  $\alpha \leq x < \alpha + \varepsilon$ .

$\mu(F)$

---

For any  $\varepsilon > 0$ ,  $\exists x \in X$  s.t.  $x < \alpha + \varepsilon$ .

(Otherwise,  $\alpha + \varepsilon$  would be a lower bound for  $X$ .)

$$\alpha = \inf X \geq \alpha + \varepsilon \quad \checkmark$$

$\text{g.l.b } X$

$(\Omega, \mathcal{Z}^\Omega, \mu)$

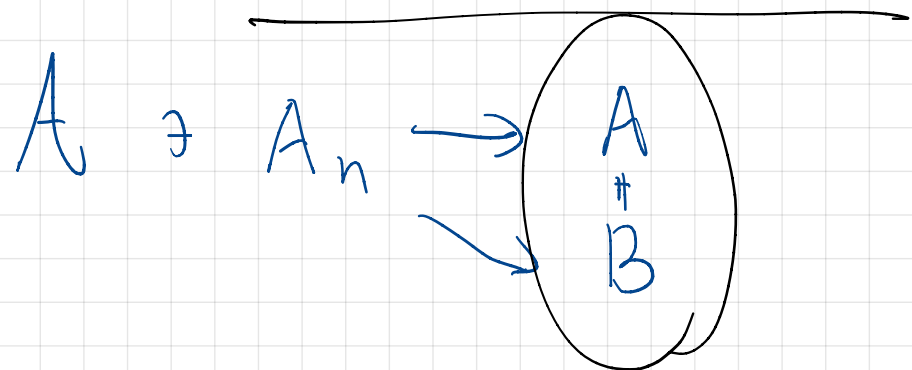
$\uparrow \downarrow \mu^* \rightsquigarrow d_\mu$

$[\mathcal{Z}^\Omega] \sim$

$d_\mu$  becomes a metric.

$E_1, E_2 \subseteq \Omega \quad E_1 \sim E_2 \iff d_\mu(E_1, E_2) = 0.$

$\forall E_1, E_2 \subseteq \overline{A} \quad \bar{\mu}(E_1 \Delta E_2) = \mu^*(E_1 \Delta E_2)$



$\mu: A \rightarrow \mathbb{R}$

$\bar{\mu}: \overline{A} \rightarrow \mathbb{R}$

$\bar{\mu}(A) := \lim_{n \rightarrow \infty} \mu(A_n)$

for any  $\{A_n\}$  s.t.  $A_n \rightarrow A$ .

$\downarrow$