Frequentism

- The SLLN justifies the "frequentist" philosophy of probability and statistics (and is the basis of all scientific experiments).
 - Given an event A, what does P(A) mean?
 - La Do repeated trials of an experiment to test A. La Record 1 if A occurs in a trial,
 - record O if A cless not occur in a trial
 - La Average the results.

More practically: the universality of the SLLN (depending only on E[X_]) makes it useful when we know little about M_Xn.

Renewal Theory

- Eg {X,3, ind L' random variables with X,20,& P(X,>0)>0
 - · Lifetime of lightbulb #n

 $S_n = \chi_1 + \chi_2 + \dots + \chi_n$

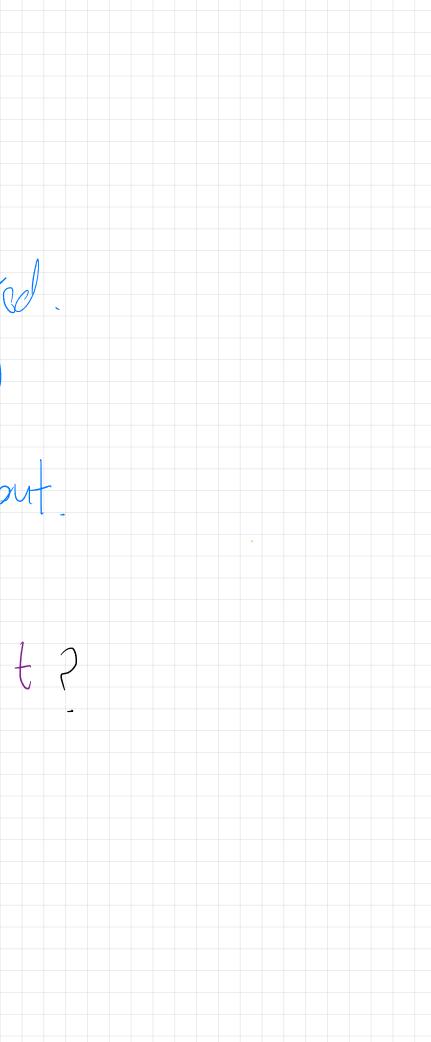
- · All the same design, manufacture, so identically distributed.
- The time @ which bulb n burns out is not influenced by the lifetimes of the other bulbs - independent.
- · Assume we replace each bulb the instant it burns out

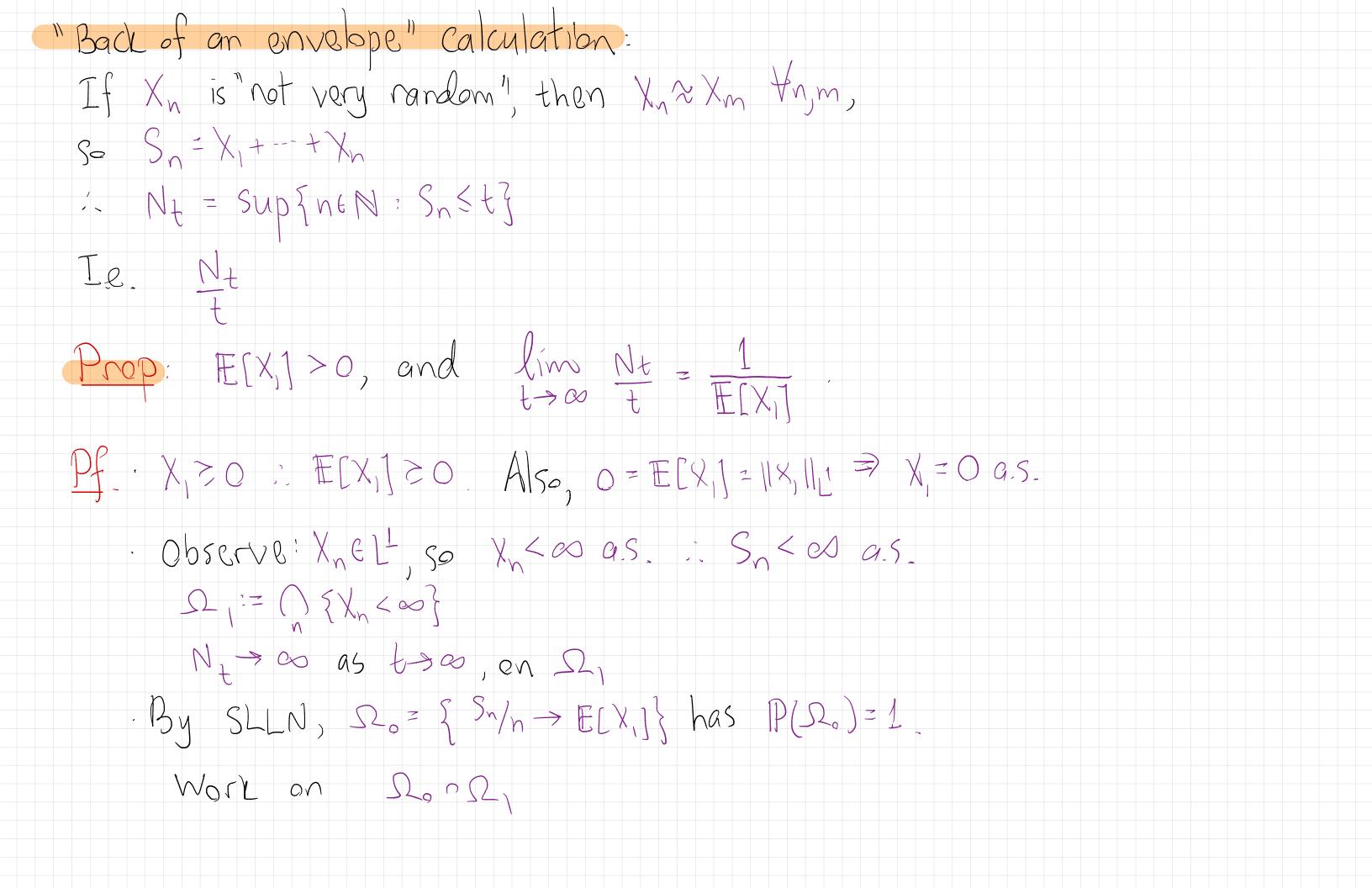
Question: How many bulbs do we need to last for time t?

Nt:=

random integer determined by

Answerable question: how does Nt behave as t->00?





By definition,



More Realistic Example

Lightbulbs burn out after independent id times Xn=0 After each bulb burns out, there's a waiting time Yn=0 before it is replaced.

 $\{\chi_{n}\}_{n=1}^{\infty}$ id $[1, \xi_{n}]_{n=1}^{\infty}$ id $[1, \xi_{n}]_{n=1}^{\infty}$ id $[1, \xi_{n}]_{n=1}^{\infty}$

 $P(X_n>0)>0$, $P(Y_n>0)>0$

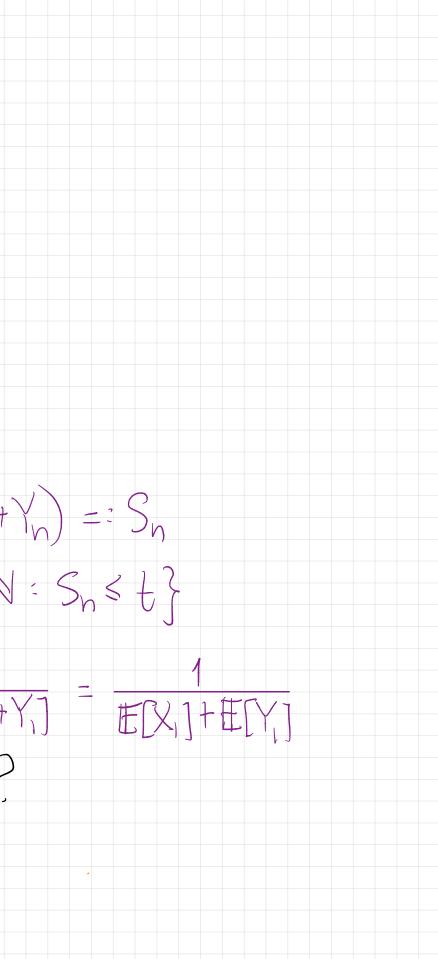
Time until $(n+1)^{st}$ bulb is replaced = $(X_1+Y_1) + \dots + (X_n+Y_n) =: S_n$ Number of bulbs needed through time $t := N_t = \sup\{n \in \mathbb{N} : S_n \le t\}$

From the last example, we know $\lim_{t \to \infty} \frac{N_t}{t} = \frac{1}{E[X_1 + Y_1]} = \frac{1}{E[X_1] + E[Y_1]}$

Question: What fraction of the time is there light?

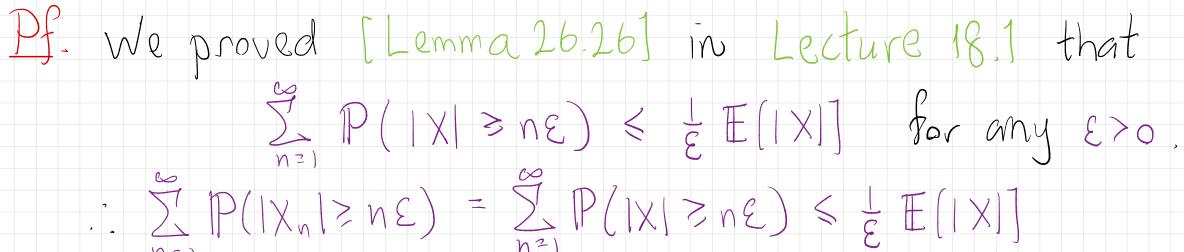
"Back of an envelope" calculation: Time with light = X, + --+ Xn

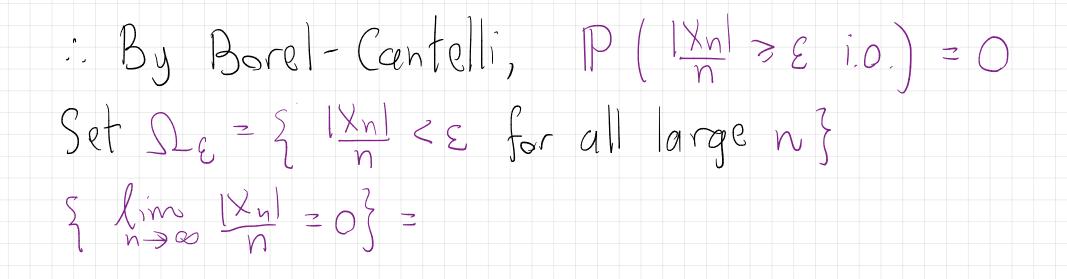
Total time = X, +Y, + - + X, +Y,

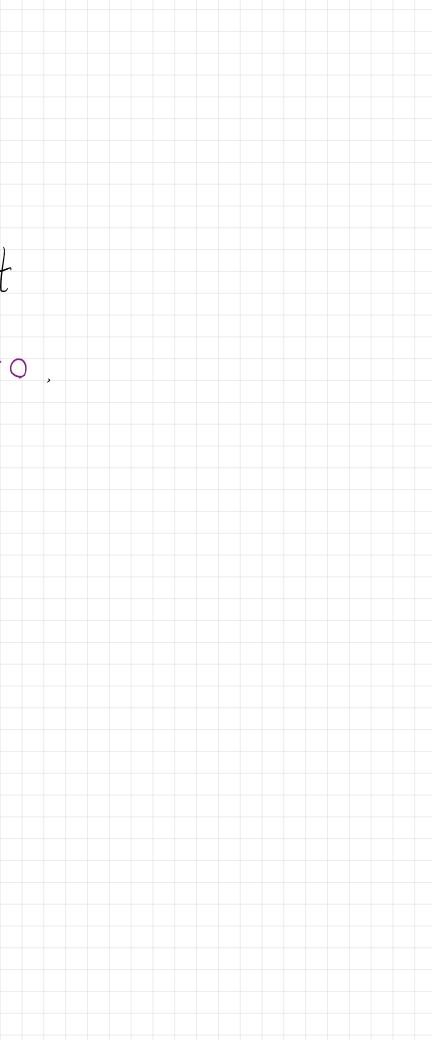




If $X \in L^1$ and $\{X_n\}_{n=1}^{\infty}$ are id with $X_n \stackrel{d}{=} X$, then $X_n \rightarrow O$ q.s.

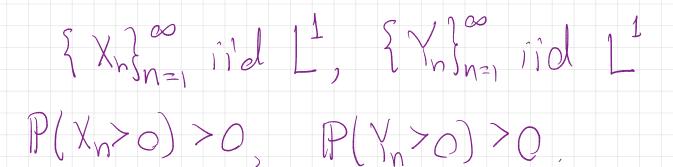


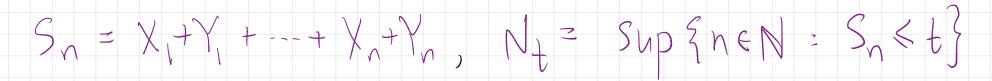




Back to the "realistic" lightbulb problem:

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Tt = length of time in [0,1] that a working lightbulb is installed.



