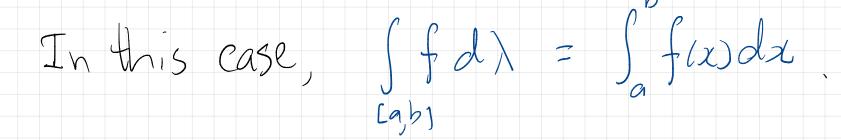
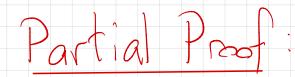


So we can integrate non-Riemann integrable things

The [11.5] Let $\overline{\mathcal{B}}$ denote the completion of $\mathcal{B}(\mathbb{R})$ with λ . Then a bounded function $f: (q,b) \to |\mathbb{R}$ is Riemann integrable iff it is $\overline{\mathcal{B}}/\mathcal{B}$ measurable, and $\lambda \{\chi \in (q,b)\}: f$ is discontinuous ($\mathfrak{D} \neq \chi \} = 0$.





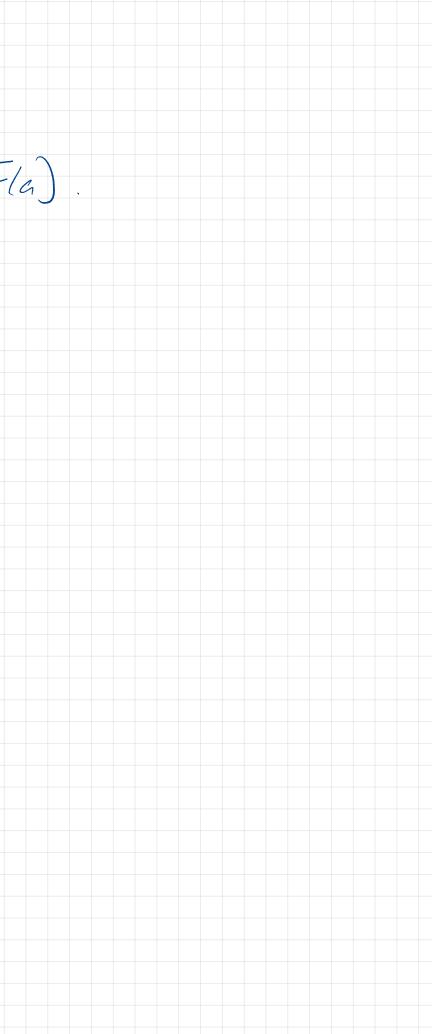


The Lebesgue integral also allows us to hendle "improper integrals". Eg. j flada := lino j flada

what about other Borel measures on R? So long as MC-nn1 < 00 the N, we mean

Radon measures, and so $\mu = \mu_F$ i.e. $\mu(a,b] = F(b) - F(a)$.

Fis right-continuous. Suppose that Fis



Of course, not every Radon measure has a density. Eq. $M = S_x = M_F$ with F = I(x, co). We see that F had better be continuous (i.e. /1F has no point mass) if we want MF to possess a density. To minic the calculations on the last page, we may not need FGC², but we at least need the Fundamental Theorem of Calculus to hold

Ingeneral: if F is continuous, differentiable a.e., and nice enough that

$F(x) = \int_{(a,x)} F' d\lambda \quad \text{for } \lambda - a.e. x$

then we can minis the preceding to see that

$M_F(A) = \int_A F' dA$

If F & C¹, this is fine. It works much more generally - but it doesn't always work, even if F is continuous, and diffible a.e.

Eq. The Devil's Stancase F diffible q.e., F'=0.