Name: Sol Utions

PID: A∅

Seat Number: 1

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MATH 180A (B00) PRACTICE EXAM 2a 11/20/19

INSTRUCTIONS — READ THIS NOW

- Write your name, PID, and current seat number, and write out the academic integrity pledge in the indicated space above. Also, write your name and PID on **every page**. Your score **will not be recorded** if any of this identifying information is missing, or if the academic integrity pledge is illegible.
- Write within the **boxed areas**, or your work will not be graded. If you need more space, raise your hand. To receive full credit, your answers must be neatly written and logically organized.
- To ask questions during the exam, remain in your seat and raise your hand. Please show your ID to a proctor when you hand in your exam.
- You may not speak to any other student in the exam room while the exam is in progress (including after you hand in your own exam). You may not share **any information** about the exam with any student who has not yet taken it.
- Turn off and put away all cellphones, calculators, and other electronic devices. You may not access any electronic device during the exam period. You may use your one double-sided page of hand-written notes, but no other books, notes, or aids.
- This is a 50-minute exam in design, but you have up to 2 hours to complete it. There are 5 problems on 5 pages, worth a total of 50 points. Read all the problems first before you start working on any of them, so you can manage your time wisely.
- **Have fun!**

Name: Sol Utions PID: A$\emptyset$ **(P. 3)**

1. Suppose that the time it takes for you to complete your probability homework is distributed according to an exponential random variable with mean 1 hour. You start your homework at 8:00 PM. Your bedtime is 10:00 PM. If you finish your homework before your bedtime, you watch TV until your bedtime and then go to sleep. If you do not finish by your bedtime, you go to sleep anyway, and so you do not watch TV at all. Let Y be the random variable that measures the amount of time in hours that you spend watching TV.

(a) (6 points) Calculate the CDF of Y.

X = time to finish HW $\sim \mathrm{Exp}(1)$

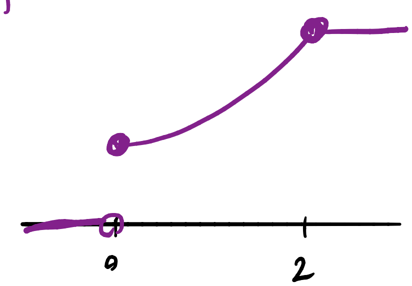
$$Y = \max(2-X, 0) = \begin{cases} 2-X & \text{if } X \in [0,2] \\ 0 & \text{if } X \geq 2 \end{cases}$$

$$F_Y(t) = \mathbb{P}(Y \leq t) = \begin{cases} 0 & \text{if } t<0 \\ \mathbb{P}(X \geq 2-t) & \text{if } t \in [0,2] \\ 1 & \text{if } t > 2 \end{cases}$$

$X \sim \mathrm{Exp}(1)$

$\mathbb{P}(X \geq s) = e^{-s}$

$$= \begin{cases} 0 & \text{if } t<0 \\ e^{t-2} & \text{if } t \in [0,2] \\ 1 & \text{if } t>2 \end{cases}$$



(b) (4 points) Calculate the expected value $\mathbb{E}[Y]$.

$Y = g(X)$ where $g(t) = \max(2-t, 0)$

$$\therefore \mathbb{E}(Y) = \int_{-\infty}^{\infty} g(t) f_X(t)\,dt = \int_0^{\infty} \max(2-t,0)\, e^{-t}\,dt$$

$$= \int_0^2 \underbrace{(2-t)}_{u}\, \underbrace{e^{-t}\,dt}_{dV}$$

$$= (2-t)(-e^{-t})\Big|_0^2 - \int_0^2 (-e^{-t})(-dt) = 2 - \int_0^2 e^{-t}\,dt = 2 + e^{-t}\Big|_0^2 = 1 + e^{-t}$$

Name: Sol Utions PID: Aϕ **(P. 5)**

2. (10 points) Let $X \sim \text{Poisson}(\lambda)$. Compute

$$\mathbb{E}\left[\frac{1}{1+X}\right]. \qquad g(t) = \frac{1}{1+t}$$

$$\mathbb{E}(g(X)) = \sum_k g(k)\,\mathbb{P}(X=k)$$

$$= \sum_{k=0}^{\infty} \frac{1}{k+1} \cdot e^{-\lambda} \frac{\lambda^k}{k!} = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{(k+1)!}$$

$$= e^{-\lambda} \cdot \frac{1}{\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{(k+1)!}$$

$$= e^{-\lambda} \cdot \frac{1}{\lambda} \sum_{\ell=1}^{\infty} \frac{\lambda^\ell}{\ell!}$$

$$= e^{-\lambda} \cdot \frac{1}{\lambda}\left(e^{\lambda} - 1\right)$$

$$= \frac{1 - e^{-\lambda}}{\lambda}$$

Name: Sol Utions PID: Aϕ **(P. 7)**

3. (10 points) Suppose that we plan to interview n randomly chosen individuals to estimate the unknown fraction $p \in (0,1)$ of the population that likes ice cream. Let $\hat{p} = \frac{S_n}{n}$ be the random variable that records the proportion of the individuals who say they do like ice cream. How many people must we interview to have at least a 95% chance of capturing the true fraction p with a margin of error .01? You may leave your answer in terms of the inverse Φ^{-1} of the CDF of the standard normal.

We know that, in general (up to the normal approximation),

$\varepsilon = 0.01$

$$\mathbb{P}(|\hat{p} - p| < \varepsilon) \geq 2\Phi(2\varepsilon\sqrt{n}) - 1 \geq 0.95$$

want

$\therefore$ Want $\Phi(0.02\sqrt{n}) \geq \frac{1}{2}(1+0.95) = 0.975$

$\therefore$ Want $0.02\sqrt{n} \geq \Phi^{-1}(0.975) = 1.96$

So

$$n \geq \left(\frac{\Phi^{-1}(0.975)}{0.02}\right)^2 = 4900$$

Name: Sol Utilons PID: Aϕ

4. Suppose that the random variable X has p.d.f.

$$f(x) = \frac{\lambda}{2} e^{-\lambda|x|}, \qquad \text{for some } \lambda > 0.$$

(a) (5 points) Compute the moment generating function $M_X(t)$ of X.

$$M_X(t) = \mathbb{E}(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \frac{\lambda}{2} e^{-\lambda|x|}\,dx = \frac{\lambda}{2}\left[\int_{-\infty}^{0} e^{tx} e^{\lambda x}\,dx + \int_0^\infty e^{tx} e^{-\lambda x}\,dx\right]$$

$$= \frac{\lambda}{2}\left[\underbrace{\int_{-\infty}^{0} e^{(t+\lambda)x}\,dx}_{=\infty \text{ if } t+\lambda \le 0} + \underbrace{\int_0^\infty e^{(t-\lambda)x}\,dx}_{=\infty \text{ if } t-\lambda \ge 0}\right]$$

only finite if $-\lambda < t < \lambda$

$$= \frac{\lambda}{2}\left[\frac{1}{t+\lambda} e^{(t+\lambda)x}\Big|_{x=-\infty}^{0} + \frac{1}{t-\lambda} e^{(t-\lambda)x}\Big|_{x=0}^{\infty}\right]$$

$$= \frac{\lambda}{2}\left(\frac{1}{t+\lambda} - \frac{1}{t-\lambda}\right) = \begin{cases} \dfrac{\lambda^2}{\lambda^2 - t^2} & |t| < \lambda \\ \infty & |t| \ge \lambda \end{cases}$$

(b) (5 points) Use the moment generating function to compute the nth moment of X.

$$\frac{\lambda^2}{\lambda^2 - t^2} = \frac{1}{1 - t^2/\lambda^2} = \sum_{k=0}^{\infty} \left(\frac{t^2}{\lambda^2}\right)^k = M_X(t) = \sum_{n=0}^{\infty} \frac{\mathbb{E}(X^n)}{n!} t^n$$

Comparing coefficients:

- If n is odd, $\mathbb{E}(X^n) = 0$
- If $n = 2k$ is even, $\dfrac{\mathbb{E}(X^{2k})}{(2k)!} = \dfrac{1}{\lambda^{2k}}$

$$\therefore \mathbb{E}(X^{2k}) = \frac{(2k)!}{\lambda^{2k}}$$

Name: Solutions PID: Aϕ **(P. 11)**

5. (10 points) Let $X \sim \mathcal{N}(0,1)$. Compute the p.d.f. of the random variable $\frac{1}{1+X^2}$. You may leave your answer in terms of the density φ of the standard normal.

Start w/ CDF:

$$\mathbb{P}\left(\frac{1}{1+X^2} \le t\right) = \mathbb{P}\left(1+X^2 \ge \frac{1}{t}\right) = \mathbb{P}\left(X^2 \ge \frac{1}{t} - 1\right)$$

$= 0$ if $t<0$

(since $X^2 \ge 0$, automatic when $\frac{1}{t} - 1 \le 0$, $t \ge 1$) since $t \ge 0$

For $0 \le t \le 1$, $\mathbb{P}\left(X^2 \ge \frac{1}{t} - 1\right) = 1 - \mathbb{P}\left(X^2 < \frac{1}{t} - 1\right)$

$$= 1 - \mathbb{P}\left(-\sqrt{\tfrac{1}{t}-1} < X < \sqrt{\tfrac{1}{t}-1}\right)$$

$$= 1 - \left(2\Phi\left(\sqrt{\tfrac{1}{t}-1}\right) - 1\right)$$

$$= 2 - 2\Phi\left(\sqrt{\tfrac{1}{t}-1}\right)$$

$$\therefore f_{\frac{1}{1+X^2}}(t) = \frac{d}{dt} F_{\frac{1}{1+X^2}}(t) = \begin{cases} 0 & t<0 \\ 0 & t>1 \\ -2\frac{d}{dt}\Phi\left(\sqrt{\tfrac{1}{t}-1}\right) & 0 \le t \le 1 \end{cases}$$

$$\frac{d}{dt}\Phi\left(\sqrt{\tfrac{1}{t}-1}\right) = \Phi'\left(\sqrt{\tfrac{1}{t}-1}\right) \cdot \frac{1}{2\sqrt{\frac{1}{t}-1}} \cdot \frac{-1}{t^2}$$

$$\therefore f_{\frac{1}{1+X^2}}(t) = \frac{1}{t^2\sqrt{\frac{1}{t}-1}}\, \underbrace{\varphi\left(\sqrt{\tfrac{1}{t}-1}\right)}_{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{1}{t}-1\right)}}\, \mathbb{1}_{\{t \in [0,1]\}}$$

Name: Sol Utions PID: A ϕ **(P. 13)**

6. [**Extra**] (10 points) Show that there is no random variable X with moment generating function $M_X(t)$ such that $M_X(1) = 3$ and $M_X(2) = 4$.

Consider the random variable $Y = e^X$.

Note that

$$M_X(n) = \mathbb{E}(e^{nX}) = \mathbb{E}((e^X)^n) = \mathbb{E}(Y^n).$$

Thus $\mathbb{E}(Y) = M_X(1) = 3$

$\mathbb{E}(Y^2) = M_X(2) = 4.$

But if this is true, then

$$\mathrm{Var}(Y) = \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 = 4 - 3^2 = \underline{\underline{-5}}$$

This is impossible: variances are always ≥ 0 b/c $\mathrm{Var}(Y) = \mathbb{E}(\underbrace{(Y - \mathbb{E}(Y))^2}_{\geq 0})$.

Thus, there can be no such Y, hence no such X.

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1. (10 points) One of the nationwide TV channels wants to determine the level of support of Yuriy N. for the future presidential elections in one of the European countries. For this, they poll n randomly chosen individuals. How large should n be to get a 95% confidence interval of size 0.1 for the true value p of support of Yuriy N. among the population? [**Hint.** You can find the table of values of $\Phi(x)$ on the last page of this exam].

Confidence interval $[p-\varepsilon, p+\varepsilon]$ has size (length) 2ε.
So want $2\varepsilon = 0.1$, $\varepsilon = 0.05$

We know that, in general (up to the normal approximation),

$$\mathbb{P}(|\hat{p}-p|<\varepsilon) \geq 2\Phi(2\varepsilon\sqrt{n})-1 \geq 0.95$$

want

$$\therefore \Phi(0.2\sqrt{n}) \geq \tfrac{1}{2}(1+0.95) = 0.975$$

Lookup on table: $\Phi(1.96) = 0.975$

$$\therefore \Phi^{-1}(0.975) = 1.96$$

So need $\overbrace{0.1}^{2\varepsilon}\sqrt{n} \geq 1.96$

$$\sqrt{n} \geq \frac{1.96}{0.1} = 19.6$$

Need $n \geq (19.6)^2 = 392.16$

i.e. $n \geq 393$

Name: Sol Utions PID: A$\emptyset$ (P. 5)

2. (10 points) The probability that a person living in SD has taken the MATH 180A *Introduction to Probability* class is 10^{-3}. You interview 250 randomly chosen people. Estimate the probability that there will be at least 2 among them who took MATH 180A (you can use the fact that $e^x \approx 1+x$ for x small). Justify your choice of approximation.

Randomly chosen people, each w low prop. of having taken 180A; so N = #people $\sim$ Poisson(λ).

$\lambda = \mathbb{E}(N) = 250 \cdot 10^{-3} = 0.25.$

$$\mathbb{P}(N \geq 2) = 1 - \mathbb{P}(N<2) = 1 - (\mathbb{P}(N=0) + \mathbb{P}(N=1))$$
$$= 1 - \left(e^{-\lambda} + e^{-\lambda}\frac{\lambda^1}{1!}\right)$$
$$= 1 - (1+\lambda)e^{-\lambda}.$$

$\lambda = 0.25$ is "small" so $e^{-0.25} \approx 1 - 0.25$

$$\therefore \mathbb{P}(N \geq 2) \approx 1 - (1+0.25)(1-0.25)$$
$$= 1 - (1-(0.25)^2) = (0.25)^2 = \frac{1}{16}$$
$$= 6.25\%$$

Name: Sol Utions PID: Aϕ

3. You are waiting for the South Campus Shuttle. From experience, you know that on average they come every 12 minutes, but their arrival times are random. You get distracted playing *The Legend of Zelda: Link's Awakening* on your Nintendo Switch, and don't notice when the buses goes by.

(a) (4 points) Find the intensity λ (in units of 1/hours) of the Poisson process that models the bus arrival times.

N_t = # busses coming by t hours. $\sim$ Poisson(λt).

$\mathbb{E}(N_t) = \lambda t$.

We know on average 1 comes every 12 minutes, so on average 5/hour.

$\therefore \underbrace{\mathbb{E}(N_1)}_{\lambda \cdot 1} = 5 \quad \Big\} \therefore \lambda = 5$.

(b) (6 points) What is the probability that 3 buses go by in the first 18 minutes you're playing, and then no buses come in the following 12 minutes?

Poisson process: if $0<s<t$, $N_t - N_s$, N_s are independent

Here $s = \frac{18}{60}$ hr $= 0.3$

$t-s = \frac{12}{60}$ hr $= 0.2$

$N_s \sim$ Poisson(λs)

$N_t - N_s \sim$ Poisson$(\lambda(t-s))$

So $\mathbb{P}(N_{0.3} = 3, N_{0.5} - N_{0.3} = 0)$

$= \mathbb{P}(N_{0.3} = 3)\,\mathbb{P}(N_{0.5} - N_{0.3} = 0)$

$= e^{-5(0.3)} \frac{(5(0.3))^3}{3!} \cdot e^{-5(0.2)} = e^{-2.5} \frac{(1.5)^3}{3!} \doteq 4.6\%$

Name: Sol Utions PID: Aϕ

4. Let X be a continuous random variable with m.g.f. $M_X(t) = e^{8t^2}$.

(a) (4 points) Find the p.d.f. of X.

If $Z \sim N(0,1)$, $M_Z(t) = e^{t^2/2}$.

$\therefore$ if $\sigma > 0$, $M_{\sigma Z}(t) = \mathbb{E}(e^{t\cdot\sigma Z}) = \mathbb{E}(e^{(\sigma t)Z})$

$= M_Z(\sigma t) = e^{(\sigma t)^2/2}$

$= e^{\frac{\sigma^2}{2} t^2}$

Now $e^{8t^2} = e^{\frac{4^2}{2} t^2}$

$\therefore M_X$ is the MGF of $4Z$.

Because $M_X < \infty$ on a nbhd of 0, $\Rightarrow X \sim 4Z \sim N(0,16)$

So $f_X(x) = \frac{1}{\sqrt{2\pi}\cdot 4} e^{-x^2/2\cdot 4} = \frac{1}{4\sqrt{2\pi}} e^{-x^2/8}$

(b) (6 points) Compute the p.d.f. of the random variable $X^3 + 1$ in terms of the p.d.f. of X.

First find CDF:

$x \mapsto x^3$ is a strictly increasing bijection $\mathbb{R} \to \mathbb{R}$.

$\mathbb{P}(X^3+1 \le t) = \mathbb{P}(X^3 \le t-1) = \mathbb{P}(X \le (t-1)^{1/3})$

$\therefore f_{X^3+1}(t) = \frac{d}{dt} F_{X^3+1}(t) = \frac{d}{dt} F_X((t-1)^{1/3})$

$= F_X'((t-1)^{1/3}) \cdot \frac{d}{dt}(t-1)^{1/3}$

$= f_X((t-1)^{1/3}) \cdot \frac{1}{3}(t-1)^{-2/3}$

$= \frac{1}{12\sqrt{2\pi}} \cdot \frac{1}{(t-1)^{2/3}} \cdot e^{-\frac{1}{8}(t-1)^{2/3}}$.

5. (10 points) Let $X \sim \text{Exp}(\lambda)$. Compute the p.d.f. of the random variable $\frac{1}{X+1}$.

$$f_X(x) = \lambda e^{-\lambda x} \mathbb{1}\{x \geq 0\}$$

$$\mathbb{P}\left(\frac{1}{X+1} \leq t\right) = \begin{cases} 0, & t \leq 0 \text{ since } X \geq 0 \text{ so } \frac{1}{X+1} \geq 0 \\ 1, & t \geq 1 \text{ since } X \geq 0 \text{ so } X+1 \geq 1 \text{ so } \frac{1}{X+1} \leq 1. \end{cases}$$

So what happens when $t \in [0,1]$? $\frac{1}{t} - 1 \geq 0$

$$\mathbb{P}\left(\frac{1}{X+1} \leq t\right) = \mathbb{P}\left(X+1 \geq \frac{1}{t}\right) = \mathbb{P}\left(X \geq \frac{1}{t} - 1\right)$$

$$= \int_{\frac{1}{t}-1}^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= -e^{-\lambda x}\Big|_{\frac{1}{t}-1}^{\infty}$$

if $t \in [0,1]$

$$= e^{-\lambda\left(\frac{1}{t}-1\right)}$$

$$\therefore f_{\frac{1}{X+1}}(t) = \frac{d}{dt}(\text{" "}) = e^{-\lambda\left(\frac{1}{t}-1\right)} \cdot \left(-\lambda\left(-\frac{1}{t^2}\right)\right)$$

$$= \begin{cases} \frac{\lambda}{t^2} e^{-\lambda\left(\frac{1}{t}-1\right)} & t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Name: ______________________ PID: ____________________

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy$$

To illustrate the use of the table: $\Phi(0.36) = 0.6406$, $\Phi(1.34) = 0.9099$

	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990