MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: § 1.3 - 1.4

Next: § 2.1 - 2.2

HW 0: double check !

HW 1 due FRIDAY, 10/04

Lab 1 due MONDAY, 10/07

Screencast & video available after each lecture @ podcast.ucsd.edu

Before/After slides now available on course webpage.

Lots of active discussion on Piazza.

Combinatorics

* selecting k objects from among n, with replacement:

  \# ways =

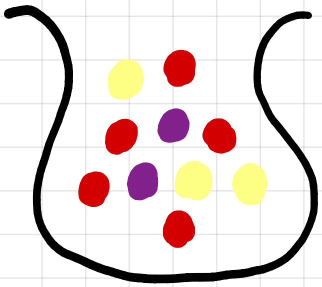
* selecting k objects from among n, without replacement; order matters:

  \# ways =

* selecting k objects from among n, without replacement; order doesn't matter:

  $$\# \text{ ways } = \binom{n}{k}$$

Sampling with Replacement (order doesn't matter)

E.g. 

An urn contains 10 balls:

2 blue

3 yellow

5 red

Problem: 3 balls are chosen without replacement.

P(2 yellow, 1 red)

1.3

What if $\#\Omega = \infty$?

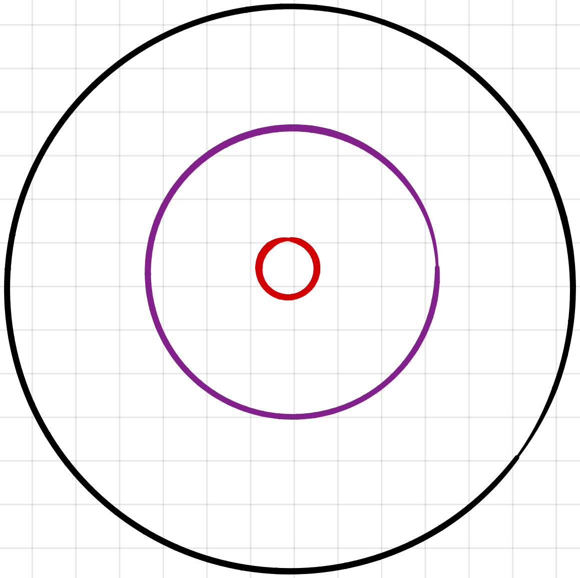
Then we need a different notion of uniform.

Eg. A random real number is chosen in $[0,1]$.

(a) What is the probability it is ≥ 0.7 ?

(b) What is the probability it is $= \frac{1}{2}$?

E.g.



An archery target is a disk 50 cm in diameter.

A blue disk in the center is 25 cm in diameter.

A red disk in the center is 5 cm in diameter.

Given that you hit the target (randomly), what are the chances of hitting the blue disk? The red disk?

Decompositions

1.4

Eg. A fair coin is tossed 5 times. What is the probability that at least 3 tosses come up tails?

E.g. A fair die is rolled 4 times. What is the probability of at least one double?

$A = \{$ some number comes up at least two times $\}$

$A_k = \{$ k comes up at least two times $\}$

$A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$

$A_k^m = \{$ k comes up exactly m times $\}$

zillions of scenarios.

$A_1 = A_1^2 \cup A_1^3 \cup A_1^4 \cup A_1^5 \cup A_1^6$

$\vdots$

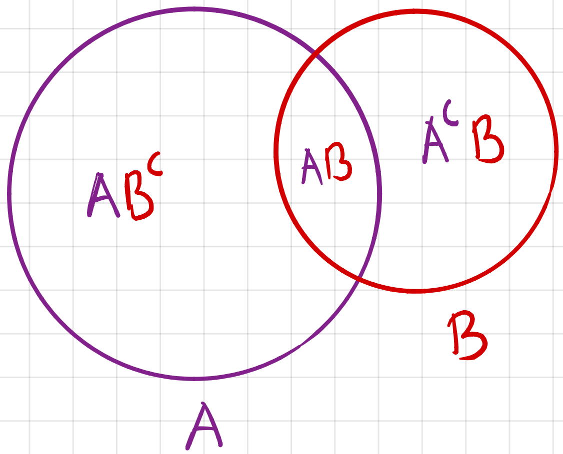
Question: Are all these events disjoint?

Sometimes, you can't avoid lack of disjointness so easily. You have to take intersections into account.

Notation: $A \cap B = \{\text{all outcomes in both } A \text{ and } B\}$

$$\parallel$$

$$AB$$

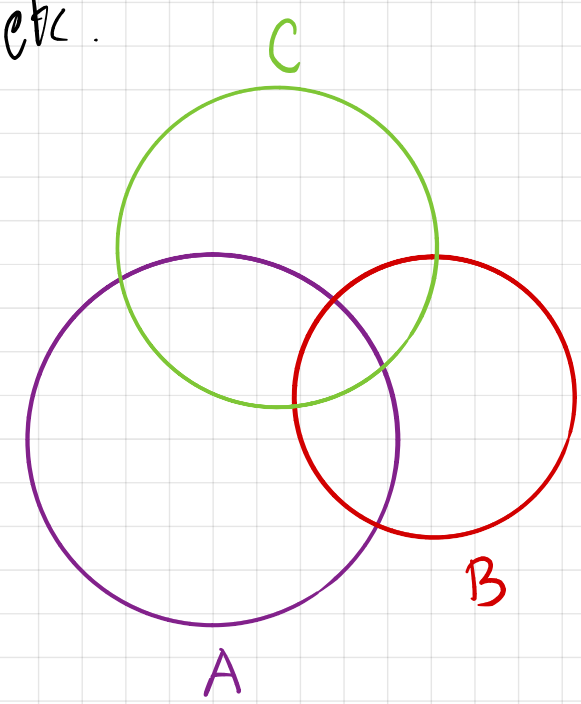


$A \cup B = AB^c \cup AB \cup A^cB$ ← disjoint

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$$

Principle of Inclusion / Exclusion

The probability of a union can be computed by adding the probabilities, then subtracting off the intersection(s) overcounted. If you have more sets, you have to keep going and re-add back in pieces that you over-subtracted, etc.

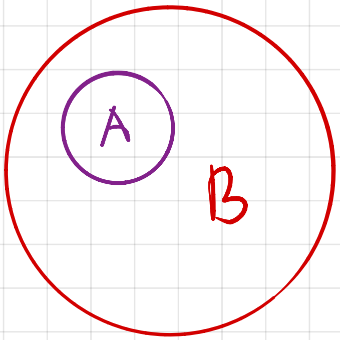


E.g. 20% of the population own cats.
25% of the population own dogs.
5% of the population own both.
What is the probabality that a random person owns neither?

Monotonicity

If $A \subseteq B$ then $B = A \cup A^c B$ is a disjoint union

$$\therefore \mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(A^c B)$$



Eg 90% of your friends like the xiao long bao at Din Tai Fung.
80% of your friends like the xiao long bao at Shanghai Saloon.
what is the smallest possible proportion of your friends who like the xiao long bao at both restaurants?