MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Homework 8 Due TODAY

Final Exam: Monday, Dec 9, 11:30a–2:30p
in REC GYM

* Bring Student ID
* Seat Assignment on TritonEd (this weekend)
* 2 double-sided sheets of hand-written notes
* If possible, eat an early lunch.

Regrade requests (for final exam & HW8) NEXT QUARTER.

It is your responsibility to make sure all your grades are in order BEFORE NEXT FRIDAY.


Please fill out
your
CAPEs !

1. Let X and Y be continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{2} + \frac{y}{4}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) (5 points) Compute $\mathbb{P}(X > 2Y)$.

1. Let X and Y be continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{2} + \frac{y}{4}, & 0 \leq x \leq 1,\ 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(b) (5 points) What is the marginal density f_X of X?

1. Let X and Y be continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{2} + \frac{y}{4}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(c) (5 points) Are X and Y independent? Explain.

2. An urn contains 5 balls: two balls labeled 1, two balls labeled 2, and one ball labeled 3. You sample 2 balls from the bin, without replacement; the number on the first one is X, and the number on the second one is Y.

(a) (5 points) Compute the joint probability mass function of X and Y. (You may find it convenient to express it in the form of a chart.)

2. An urn contains 5 balls: two balls labeled 1, two balls labeled 2, and one ball labeled 3. You sample 2 balls from the bin, without replacement; the number on the first one is X, and the number on the second one is Y.

(b) (5 points) Compute the probability mass function of X and the probability mass function of Y.

2. An urn contains 5 balls: two balls labeled 1, two balls labeled 2, and one ball labeled 3. You sample 2 balls from the bin, without replacement; the number on the first one is X, and the number on the second one is Y.

(c) (5 points) Are X and Y independent? Justify your answer.

3. Let $U_1, U_2, \ldots, U_n, \ldots$ be independent, identically distributed random variables, each with the $\mathrm{Uniform}[-2, 2]$ distribution. Let $S_n = U_1 + U_2 + \cdots + U_n$.

(a) (5 points) Compute $\mathbb{E}(S_n)$ and $\mathrm{Var}(S_n)$.

3. Let $U_1, U_2, \ldots, U_n, \ldots$ be independent, identically distributed random variables, each with the $\mathrm{Uniform}[-2, 2]$ distribution. Let $S_n = U_1 + U_2 + \cdots + U_n$.

(b) (5 points) For any $\epsilon > 0$, what can you say about

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{|S_n|}{n^{2/3}} \geq \epsilon\right)\ ?$$

4. Let T be the triangle in $\mathbb{R}^2$ with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$ (including the interior). Suppose that $P = (X, Y)$ is a point chosen uniformly at random inside of T.

(a) (5 points) What is the joint density function of (X, Y)? Use this to compute $\operatorname{Cov}(X, Y)$.

4. Let T be the triangle in $\mathbb{R}^2$ with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$ (including the interior). Suppose that $P = (X, Y)$ is a point chosen uniformly at random inside of T.

(b) (5 points) Determine if X and Y are independent.

5. Suppose $X_1, X_2, \ldots, X_n, \ldots$ are i.i.d. random variables with mean $\mathbb{E}(X_j) = 0$ and variance $\mathrm{Var}(X_j) = 1$. Determine the following limits with precise justifications.

(a) (5 points) $\lim\limits_{n\to\infty} \mathbb{P}\left(-\frac{n}{4} \leq X_1 + \cdots + X_n < \frac{n}{2}\right)$

5. Suppose $X_1, X_2, \ldots, X_n, \ldots$ are i.i.d. random variables with mean $\mathbb{E}(X_j) = 0$ and variance $\mathrm{Var}(X_j) = 1$. Determine the following limits with precise justifications.

(b) (5 points) $\lim_{n\to\infty} \mathbb{P}(X_1 + \cdots + X_n = 0)$

Please fill out your CAPEs !

And...

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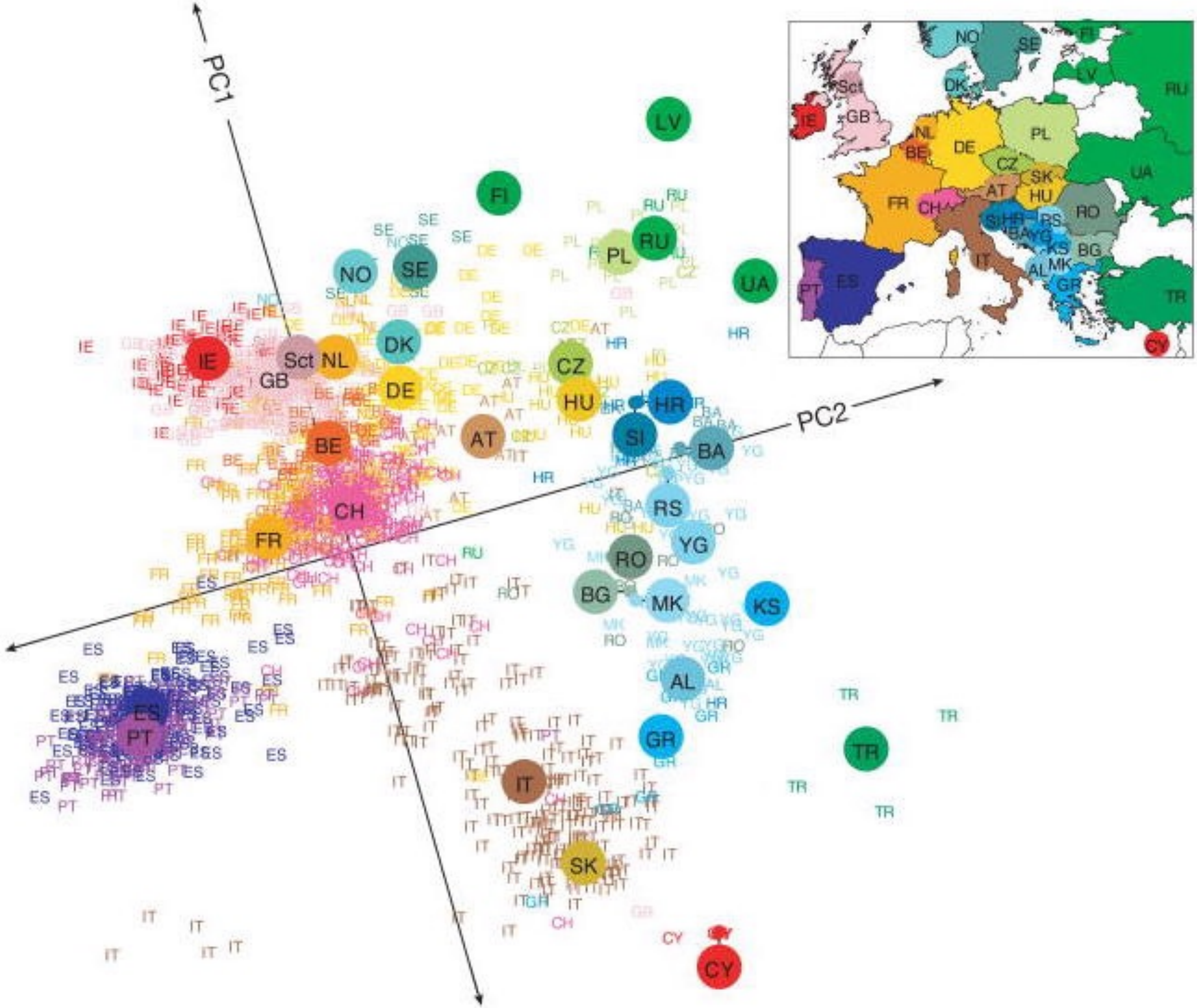
Hidden Data in Random Matrices



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