MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: REVIEW

Next: REVIEW

CAPEs ! Please fill them out.

Lab 7 Due TODAY.

Homework 8 Due Friday, Dec 6.

Final Exam: Monday, Dec 9, 11:30a–2:30p
in REC GYM

Extra/Extended Office Hours

Thursday
8:30 – 10:00am

Friday
10:00 – 11:30am

Main Topics Covered before Midterm 2

* Counting propability
* Conditional probability & Bayes' Rule
* Independence of Events
* Random variables & distributions
* CDF, PMF / PDF
* Independent trials & sampling
* $\mathbb{E}$ & Var
* Chebyshev's Inequality
* Ber, Bin, Geom, Poisson, $\mathcal{N}$, Exp, ...
* Normal Approximation & Confidence intervals
* Poisson Approximation
* Poisson Process
* MGF

Moment Generating Functions

Random Variable X $\rightsquigarrow$ MGF $M_X : \mathbb{R} \to (0, \infty]$

$$M_X(t) = \mathbb{E}(e^{tX})$$

Some important examples: $X \sim \mathcal{N}(0,1)$, $Y \sim Exp(\lambda)$, $N \sim Poisson(\lambda)$

$$M_X(t) = e^{t^2/2}$$

$$M_Y(t) = \frac{\lambda}{\lambda - t}, \; t < \lambda \quad (\infty \text{ otherwise})$$

$$M_N(t) = e^{\lambda(e^t - 1)}$$

Theorem: If $M_X(t) < \infty$ for all $t \in$ some open interval including 0, then the function M_X <u>uniquely</u> determines the distribution of X.

I.e.

E.g. (5.17) $f_X(x) = \begin{cases} x/2 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

(a) Find $M_X(t)$. (b) Use it to compute $E(X^n)$ for all n.

Eg. (5.18) Let $X \sim Geom(p)$. Compute $M_X(t)$.
Use it to compute $\mathbb{E}(X)$ and $Var(X)$.

MGF and Independent Sums

If $X_1, X_2, \dots, X_n$ are independent, and $S_n = X_1 + \cdots + X_n$, then

$$M_{S_n}(t) = \mathbb{E}(e^{tS_n}) =$$

E.g. Compute the MGF of a $Bin(n,p)$ random variable.
Use it to compute the mean and variance.

Eg. (8.13) Suppose $M_Z(t) = \left(\frac{1}{2}e^{-t} + \frac{2}{5} + \frac{1}{10}e^{t/2}\right)^{36}$.

Show that Z can be written as a sum of i.i.d. random variables X_j, and describe the distribution of X_j.

E.g. Let $U \sim \text{Unif}[-1,1]$, $X \sim N(1,1)$, independent.

Find the MGF of $U+X$.