MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: §5.2, 6.1

Next: §6.2-6.3

Lab 5 due TONIGHT by 11:59pm

HW 6 due Friday (Nov 15) by 11:59pm

Midterm 2: Next Wednesday ← CENTR 101!
covering Chapters 3-6.

Why Should I Care About $M_X(t)$?

Theorem: Suppose $M_X(t) < \infty$ for all t in some neighborhood $(-\varepsilon, \varepsilon)$ of 0.

Then the function M_X uniquely determines the distribution of X.

(I.e. you can recover F_X from M_X.)

F_X, f_X, p_X, M_X different tools to use in different contexts.

5.2

Let $X \sim \mathcal{N}(0,1)$.

Question: what is the distribution of X^2?

↳ To begin: which tool should we use? F_{X^2}? f_{X^2}? p_{X^2}? M_{X^2}?

Eg. Toss a fair die, yielding $X \in \{1,2,3,4,5,6\}$.
What is the probability distribution of $|X-3|$?

In general: if X is discrete, so is $g(X)$, and

$$\mathbb{P}(g(X)=t) =$$

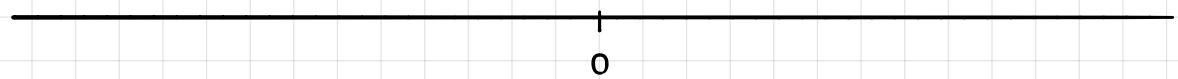
Important Example

Let X be a random variable.
What is the distribution of $Y = F_X(X)$?

Question: How does a computer generate a $\mathcal{N}(0,1)$ random variable?

To begin: we assume there is a way to produce a $U \sim \text{Unif}([0,1])$ random sample.

Eg. The CDF of $\text{Exp}(\lambda)$ is $F(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-\lambda t}, & t \geq 0 \end{cases}$.



Question

6.1

Suppose X and Y are both $\mathrm{Ber}(p)$ random variables. What is $\mathbb{P}(X=Y)$?

(a) $p \cdot p + (1-p)\cdot(1-p)$

(b) $p\cdot(1-p) + (1-p)\cdot p$

(c) 0

(d) 1

(e) Not enough information.

Definition: Given (discrete) random variables $X_1, X_2, \dots, X_n$ all defined on the same sample space, their **joint distribution** is the collection of all

$$\left\{ \begin{array}{l} \mathbb{P}(X_1 = k_1, X_2 = k_2, \dots, X_n = k_n) \\ \text{all possible values } k_1 \text{ of } X_1,\ k_2 \text{ of } X_2, \dots, k_n \text{ of } X_n \end{array} \right\}$$

Eg. $X, Y \sim \mathrm{Ber}(p)$,

		X	
		0	1
Y	0		
	1		

Recovering X_j from $\underline{X} = (X_1, X_2, \ldots, X_n)$: <u>Marginals</u>

Suppose we know $p_{\underline{X}}(\underline{k})$ for all $\underline{k} = (k_1, k_2, \ldots, k_n)$.
How can we find $p_{X_1}(t)$?