MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: § 1.2 – 1.3

Next: § 1.4 – 2.1

HW.0 due TONIGHT!
HW.1 due FRIDAY, 10/04

Lab.1 due MONDAY, 10/07

Sections / Labs:

B01, B02, B03, B04 : ~~CENTR 207~~ → ERC 117

B05, B06 : ~~HSS 1128A~~ → CSB 115

Mathematical thinking

Probability Space $(\Omega, \mathcal{F}, \mathbb{P})$

Determined by the ________ being ________

Key property: if $\underbrace{A_1, A_2, A_3, \ldots}_{\in \mathcal{F}}$ are **disjoint** $(A_i \cap A_j = \phi \ \ i \neq j)$

then $\mathbb{P}(A_1 \cup A_2 \cup A_3 \cup \cdots) = \sum_j \mathbb{P}(A_j)$

Important special case: if $\#\Omega < \infty$ then

$\Omega = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_n\}$

the singleton sets $\{\omega_1\}, \{\omega_2\}, \ldots, \{\omega_n\}$ are disjoint

$\therefore \mathbb{P}(A) =$

$A = \{a_1, a_2, \ldots, a_r\}$

Uniform Probability Measure (Sampling)

1.2

If Ω is finite, the **uniform probability measure** is defined by:

For each $\omega \in \Omega$, $\mathbb{P}(\omega) =$

$\Rightarrow$ For any event A, $\mathbb{P}(A) =$

This means calculating probabilities in such models is tantamount to ________________ .

E.g. A fair die is cast 2 times. What is the probability that the sum is 4?

E.g. A fair coin is tossed 3 times.

$A = \{\text{at least two tails}\}$

$B = \{\text{exactly two tails}\}$

THINK PAIR SHARE

There are 10 people on a committee.
How many different ways are there to select a subcommittee of 4 people?

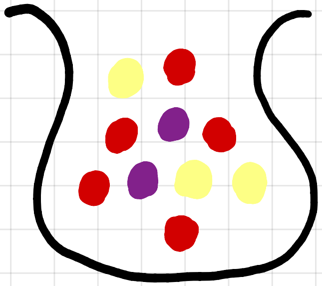
(a) $10 \cdot 10 \cdot 10 \cdot 10 = 10^4 = 10{,}000$

(b) $10 \cdot 9 \cdot 8 \cdot 7 = 5{,}040$

(c) $\binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} = 210$

(d) $\frac{10!}{4!} = 151{,}200$

Combinatorics



A collection of labeled balls $\{1, 2, \ldots, n\}$ are in an urn. k are taken out one by one

* with replacement, or
* without replacement (k)

They are lined up

* in the order they came out, or
* disregarding order.

How many ways?

* with replacement :

* without : { & ordered :

  not :

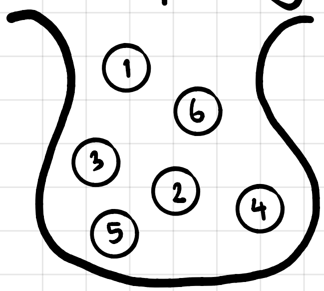
when $k=n$
$\Downarrow$

Sampling with Replacement

Toss a fair coin n times; record a statistic observing # H vs. # T.

E.g. $n = 10$, $\mathbb{P}\{\text{odd rolls are all } H\}$.

Sampling without Replacement (order matters)

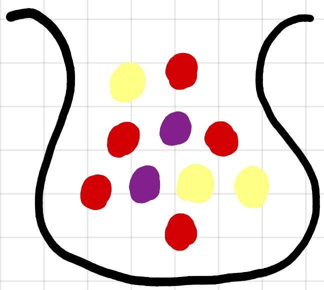


There are 6 labeled balls in an urn. 3 are removed in sequence (without replacement), and lined up in order.

What is the probability that the first two are $(3, 6)$?

Sampling with Replacement (order doesn't matter)

E.g.



An urn contains 10 balls:

2 blue
3 yellow
5 red

Problem: 3 balls are chosen without replacement.

P(2 yellow, 1 red)

1.3

What if $\#\Omega = \infty$?

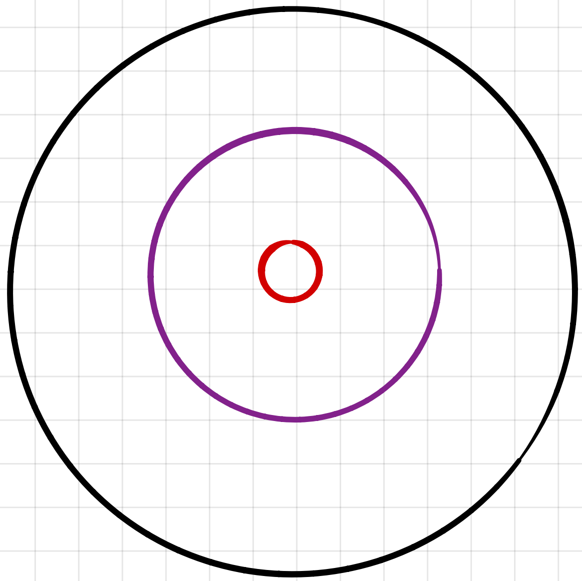
Then we need a different notion of uniform.

Eg. A random real number is chosen in $[0,1]$.

(a) What is the probability it is ≥ 0.7?

(b) What is the probability it is $= \frac{1}{2}$?

E.g.



An archery target is a disk 50 cm in diameter.

A blue disk in the center is 25 cm in diameter.

A red disk in the center is 5 cm in diameter.

Given that you hit the target (randomly), what are the chances of hitting the blue disk? The red disk?