# MATH 180A: INTRO TO PROBABILITY
## (FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: §5.1-5.2

Next: §6.1-6.2

Lab 5 due Wednesday (Nov 13) by 11:59 pm

HW 5 due TONIGHT by 11:59 pm

→ Error in problem statement of Exercise 4.40: the exact probability is $\doteq 0.00327556$

$\neq 0.000949681$

# Functions to Describe Probability Distributions

5.1

Random variable $X$.

* CDF $F_X(t) = \mathbb{P}(X \leq t)$

* PMF $p_X(k) = \mathbb{P}(X = k)$

* PDF $f_X(t) = \frac{d}{dt} F_X(t)$

New entry: MGF

$M_X(t) =$

Eg. $X \sim \mathrm{Ber}(p)$

Eg. $N \sim \mathrm{Poisson}(\lambda)$

Eg. $Z \sim \mathcal{N}(0,1)$

E.g. $T \sim \mathrm{Exp}(\lambda)$

A MGF may take some infinite values.
There is always at least one finite value:

But it can happen that there are no others!

Eg. Cauchy density $f(x) = \dfrac{1}{\pi(1+x^2)}$

# Why MGF?

Given a random variable $X$, its moments (should they exist) are the numbers $\mathbb{E}(X^k)$, $k=0,1,2,\dots$

These can be computed from $M_X(t)$ as follows.

**Theorem:** Suppose $M_X(t) < \infty$ for all $t$ in some neighborhood of $0$
$(-\varepsilon, \varepsilon)$ $\varepsilon > 0$.

Then $M_X$ is analytic on this neighborhood: its Taylor series based @ $0$ converges to $M_X(t)$ on this interval, and

$$M_X(t) = \sum_{k=0}^{\infty}$$

Eg. Find the moments of the $\text{Exp}(\lambda)$ distribution.

Eg. Find the moments of the $\mathcal{N}(0,1)$ distribution.