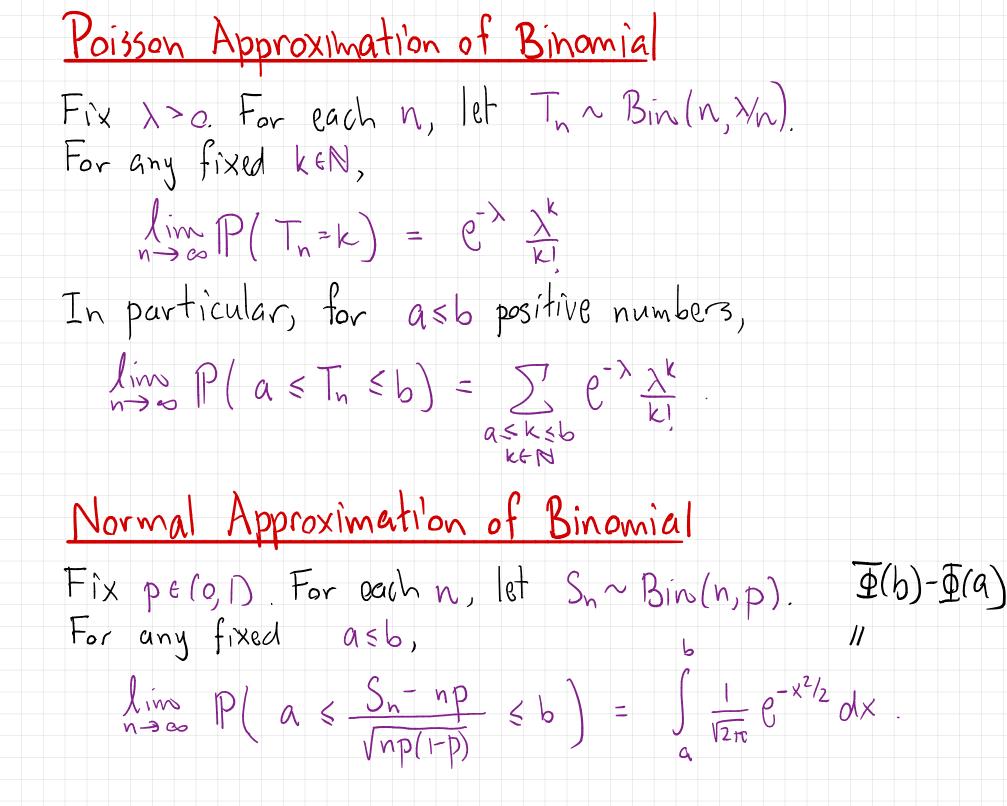
## MATH 180A: INTRO TO PROBABILITY (FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180Å

Today: §4.1-4.3 Next: §4.3-4.4

Midtern grades released; regrade requests Wed 10/30 8am-11pm Lab 3 grades released; regrade requests Thu 10/31 8am-11pm HW4 due Friday by 11:59pm Lab 4 due next Wednesday (Nov 6) by 11:59pm



Question:

A fair 20-sided die is tossed 400 times. We want to calculate the probability that a 13 came up at least 25 times. We should use:

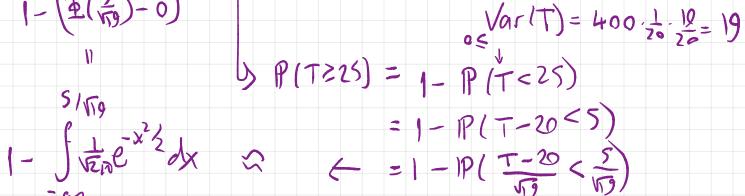
(a) Poisson Approximation. Poisson: n=400, λ=20 P(T = 25) = I - P(T < 25)(b) Normal Approximation. (c) Either  $= 1 - \sum_{k=1}^{m} P(T=k)$ (d) Neither.  $\approx 1 - \sum_{k=0}^{24} e^{-20} (20)^{k}$  $\kappa!$ 

T~ Bin (400, 1) 12.57% P(T=25)=1-P(T=24)  $= 1 - \sum_{k=0}^{1} [\frac{400}{k}] (\frac{1}{20}) (\frac{1}{10})^{k} (\frac{1}{10$ 

- 15.10%

 $\left|-\left(\overline{\Phi}\left(\frac{9}{19}\right)-0\right)\right|$ 

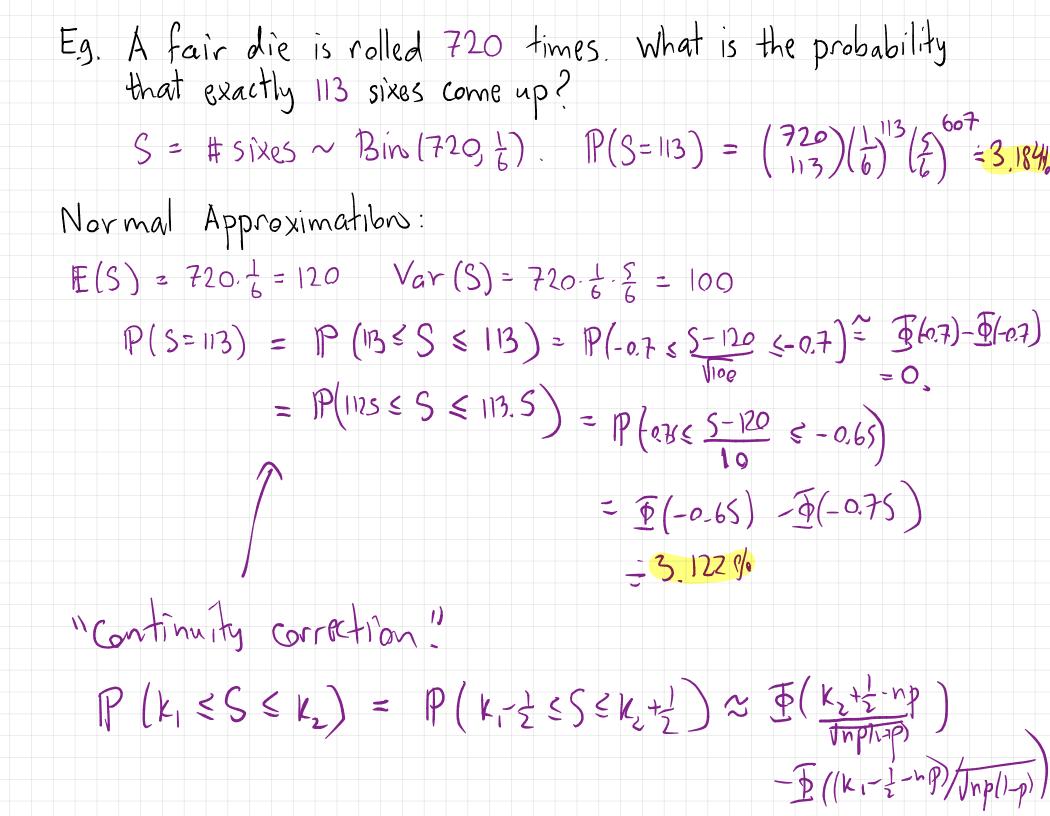
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Bin (400,1)  $Bm(n, \lambda/n)$ 

· 15.68%

Normal Approx E(T) = 400-1= 20



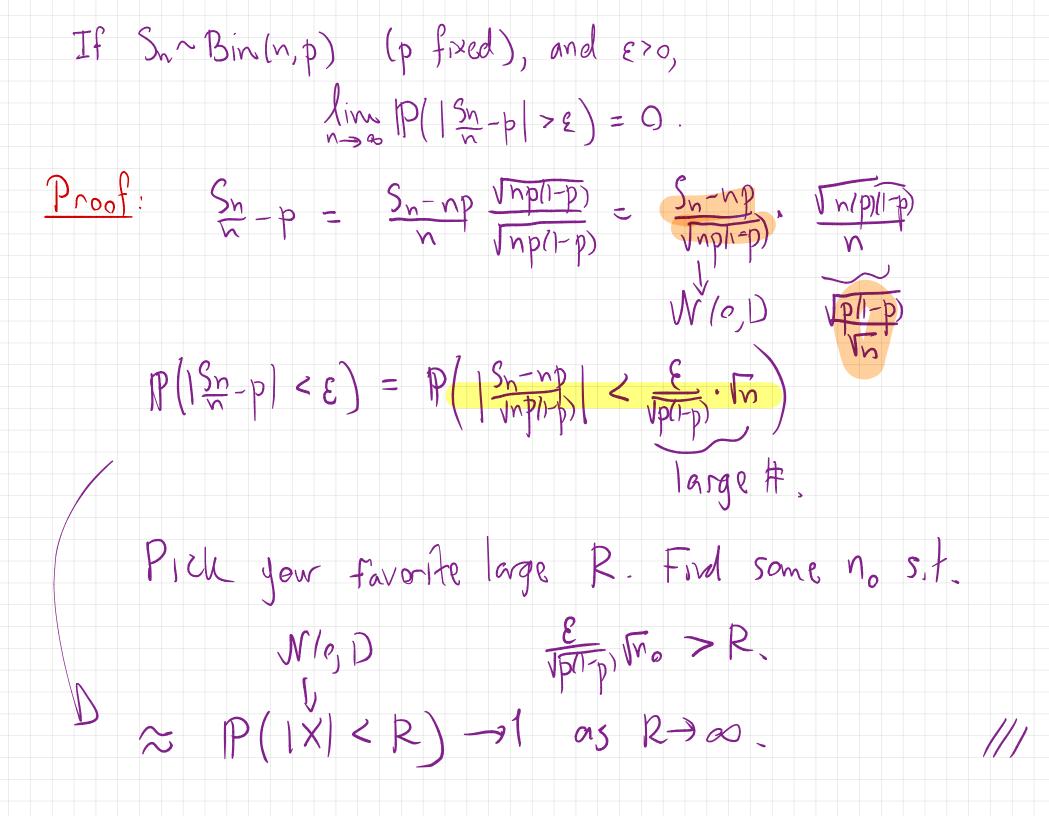
## What is E?

Precise definition:  $E(X) = \sum_{k} k \cdot P(X=k)$ Intuition: Sample X independently many times, compute te average.

Binomial  $S_n \sim Bin(n,p) = np$  $X_1 + \cdots + X_n$ ,  $X_j \sim Ber(p) E(x_j) = p$ .  $E(\sum_{n=1}^{n}) = np = p$ 

Theorem (Law of Large Numbers for Berneulli Trials) Let  $X_1 X_2 \dots X_n$  be independent Bernoulli trials with success probability p. Then " $X_1 + \dots + X_n \rightarrow p$  as  $n \rightarrow \infty$ " Precisely. For my E>O, Sulh

 $\lim_{n \to \infty} \mathbb{P}(|S_n - p| < \varepsilon) = 1$ 



Example

Flip a fair coin n times How dees

 $\lim_{n \to \infty} \mathbb{P}\left(\frac{\# \text{Heads}}{n}, 50.01\%\right) = O$ 

behave as n->00?

Suppose after 10,000 flips, there are 5,001 Heads. Should we doubt that the coins is really fair? What if, after 1,000,000 flips, there are 500,100 Hegds. Now how confident should we be that the coin is really fair?  $P(\frac{Sn}{n} = \frac{1}{2} + \varepsilon) = P(\frac{Sn - \frac{1}{2}n}{\sqrt{n \pm \frac{1}{2}}} = 2\varepsilon \sqrt{n}) \approx P(\chi = 2\varepsilon \sqrt{n})$   $\frac{1}{\sqrt{n \pm \frac{1}{2}}}$  N(e, D)10.0=3