MATH 180A: INTRO TO PROBABILITY
(FOR DATA SCIENCE)

www.math.ucsd.edu/~tkemp/180A

Today: § 3.3

Next: § 3.4

HW3 due TONIGHT!

Lab 3.2 due Monday.

* Regrade requests for HW2: window Tuesday, 10/22 8am – 11pm
  - ↳ separate request for each problem
  - ↳ detailed, polite responses please.

* Midterm 1: Wednesday, 10/23, 8-10pm, PCYNH 109
  Assigned seats (see TritonEd)
  1 double sided sheet of (hand-written) notes allowed
  Practice Midterms posted on webpage

Poisson Distribution

A random variable X has the Poisson(λ) distribution if

$$\mathbb{P}(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad k=0,1,2,\ldots$$

Eg. A 100 year storm is a storm magnitude expected to occur in any given year with probability $1/100$.
Over the course of a century, how likely is it to see at least 4 100 year storms?

Summary

Sampling independent trials, the most important (discrete) probability distributions are:

- $Ber(p)$: $\mathbb{P}(X=1)=p$, $\mathbb{P}(X=0)=1-p$ $\quad 0 \le p \le 1$

  (single trial with success probability p)

- $Bin(n,p)$: $\mathbb{P}(S_n=k)=\binom{n}{k}p^k(1-p)^{n-k}$ $\quad 0 \le k \le n$

  (number of successes in n independent trials with rate p)

- $Geom(p)$ $\quad \mathbb{P}(N=k)=(1-p)^{k-1}p$ $\quad k=0,1,2,\dots$

  (first successful trial in repeated independent trials with rate p)

- $Poisson(\lambda)$ $\quad \mathbb{P}(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$ $\quad k=0,1,2,\dots$ $\quad \lambda>0$.

  (Approximates $Bin(n,\lambda/n)$; number of rare events in many trials)

Expectation

3.3

Toss a fair coin 1000 times, and record the sequence of outcomes.

Average them.

What size do you expect this number to have?

What if the coin is biased $\mathbb{P}(X_j = 1) = p$, $\mathbb{P}(X_j = 0) = 1-p$.

Definition: Let X be a discrete random variable with possible values $t_1, t_2, t_3, \ldots$ The expectation or expected value of X is

$$\mathbb{E}(X) :=$$

Question: Is the expectation $\mathbb{E}(X)$ the value X is equal to most often?

(a) Yes, always.

(b) No, not generally.

Eg. Let X be the number rolled on a fair die.

Eg. Let Y be $\mathrm{Ber}(p)$.

Eg. You toss a biased coin (Y) repeatedly until the first heads. How long do you expect it to take?

E.g. $S_n \sim Bin(n,p)$ $(S_n = X_1 + X_2 + \cdots + X_n$ for X_j independent $Ber(p))$

$$\mathbb{E}(S_n) = \sum_{k=0}^{n} k \cdot \mathbb{P}(S_n = k) = \sum_{k=0}^{n} k \cdot \binom{n}{k} p^k (1-p)^{n-k}$$

Eg. $X \sim Poisson(\lambda)$

$$\mathbb{E}(X) =$$

↳ Eg. A factory has, on average, 3 accidents per month. Estimate the probability that there will be exactly 2 accidents this month.

E.g. Toss a fair coin until tails comes up. If this is on the first toss, you win \$2 and stop. If heads comes up, the pot doubles, and you continue. That is, if the first tails is on the k^{th} toss, you win dollars.

What is your expected winnings?

Expectation of Continuous Random Variable (with density)

X discrete, $X \in \{t_1, t_2, t_3, \dots\}$

$$\mathbb{E}(X) = \sum_j t_j \, \mathbb{P}(X = t_j)$$

X continuous ($\mathbb{P}(X=t)=0$ for each $t \in \mathbb{R}$)

propability density $f_X(t)$

Eg. Let $X \sim \mathrm{Unif}([a,b])$. $f_X(t) = \begin{cases} \frac{1}{b-a} & a \le t \le b \\ 0 & \text{otherwise} \end{cases}$

Question:

Shoot an arrow at a circular target of radius 1. What is the expected distance of the arrow from the center?

(a) 1

(b) 2/3

(c) 1/2

(d) 1/4

E.g. $f(t) = \begin{cases} 0 & t \leq 1 \\ 1/t^2 & t > 1 \end{cases}$ is a probability density.

If X is a continuous random variable with $f_X = f$, what is $\mathbb{E}(X)$?