MATH 180A: INTRO TO PROBABILITY (FOR DATA SCIENCE)

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- ↳ Instructional team
- ↳ Course calendar (Google)
- ↳ Lecture schedule
- ↳ Homework → HW.0 due MONDAY, 09/30; HW.1 due FRIDAY, 10/04
- ↳ Datahub Labs → Lab.1 due MONDAY, 10/07
- ↳ Links to Piazza, Gradescope

Exams:

Wednesday, 10/23	8-10p	
Wednesday, 11/20	8-10p	NO MAKEUP EXAMS
Monday, 12/9	11:30a–2:29p	MUST ATTEND

THINK PAIR SHARE

There are close to 180 students in this room. What are the odds that at least two share the same birthday?

(a) VERY unlikely

(b) $\frac{180}{365} \approx 50\%$

(c) $\binom{180}{2} / \binom{365}{2} \approx 25\%$

(d) VERY likely

The world around us is fundamentally random.

* 1600s – games of chance
* 1900s – quantum theory
* 1950s – finance / insurance
* 1980s – chemical reactions inside our cells
* 2000s – complex networks; machine learning

The modern rigorous foundation of probability theory goes back to 1933 Kolmogorov

Ingredients

Sample Space Ω = the set of possible outcomes in an experiment. $\{HH, HT, TH, TT\}$

Events $\mathcal{F}$ = collections of outcomes. $E = \{\text{at least 1 } H\} = \{HH, HT, TH\}$

Probability Measure $P : \mathcal{F} \to [0, 1]$

Kolmogorov's Axioms

(i) For any event $A \subseteq \mathcal{F}$, $0 \le \mathbb{P}(A) \le 1$.

(ii) $\mathbb{P}(\Omega) = 1$ $\quad$ $\mathbb{P}(\emptyset) = 0$

(iii) If $A_1, A_2, A_3, \ldots$ are disjoint events, then

$$\mathbb{P}(A_1 \cup A_2 \cup A_3 \cup \cdots) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \mathbb{P}(A_3) + \cdots$$

$\mathbb{P}(E \cup T) = \frac{2}{3}$

$\| $

$\mathbb{P}(E) + \mathbb{P}(T)$

$\frac{1}{2} + \frac{1}{3}$

E.g. A fair die. $\Omega = \{1, 2, 3, 4, 5, 6\}$ $\quad$ $\mathbb{P}(\{1\}) = \mathbb{P}(\{2\}) = \cdots = \mathbb{P}(\{6\}) = \frac{1}{6}$

E = even, O = odd

T = divisible by 3

$E = \{2, 4, 6\}$ $\quad$ $O = \{1, 3, 5\}$

$T = \{3, 6\}$

$E \cup O = \Omega$

$E \cup T = \{2, 3, 4, 6\}$

$\mathbb{P}(E) = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}(T) = \frac{2}{6} = \frac{1}{3}$.

$\mathbb{P}(E \cup T)$

$= \mathbb{P}\{2, 3, 4, 6\}$

$= \mathbb{P}\{2\} + \mathbb{P}\{3\} + \mathbb{P}\{4\} + \mathbb{P}\{6\}$

$= 4 \cdot \frac{1}{6} = \frac{2}{3}$.

Eg. Two fair dice $\Omega = \{(i,j) : 1 \le i,j \le 6\}$ $\mathbb{P}(\{i,j\}) = \frac{1}{36}$

$D = \{\text{the sum of the two dice is } 8\}$

$= \{(4,4), (2,6), (6,2), (3,5), (5,3)\}$

$\mathbb{P}(D) = \mathbb{P}(\{4,4\}) + \mathbb{P}(\{2,6\}) + \cdots + \mathbb{P}(\{5,3\}) = \frac{5}{36}$

Uniform Probability Measure