Mathematics 31CH: "Honors Vector Calculus" Syllabus
(revised September 2016)

Lecture schedule based on:
Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fifth edition
by John H. Hubbard and Barbara Burke Hubbard.

Lecture	**Section(s)**	**Topic(s)**
1	4.1	Review the definition and properties of the Riemann integral.
2	4.5	Iterated integrals and Fubini's Theorem. It is nice to include Example 4.5.7, volumes of balls in $\mathbb{R}^n$. Skip "Computing probabilities using integrals".
3	4.8	Determinants in general (only the 2×2 and 3×3 cases were discussed in 31AH). Skip "The trace and the derivative of the determinant".
4	4.8	The characteristic polynomial and the standard approach to finding eigenvalues.
5	4.9	Determinants and volume. Linear change of variables.
6	4.10	General change of variables formula for multiple integrals. Heuristic derivation only. Polar, cylindrical, and spherical special cases.
7	5.1	Volumes of k-parallelograms in $\mathbb{R}^n$.
8	5.2	"Relaxed" parametrizations for computing volumes of manifolds.
9	5.3	Computing volumes of manifolds. Independence of parametrization.
10	5.3	Examples of length, area, and volume computations.
11	6.1	Differential forms on $\mathbb{R}^n$, elementary k-forms as basis.
12	6.1	Wedge product, form fields.
13	6.2	Integrating form fields over parametrized domains.
14	6.2, 6.3	Orientation of manifolds, especially curves and surfaces.
15	6.3	Orienting manifolds defined by equations in $\mathbb{R}^n$.
16	6.4	Orientation-preserving parametrizations. Integrating forms over oriented manifolds. Skip "A nonorientable manifold".
17	6.4	Integrating forms: examples.
18	6.5	Dictionary between forms and vector fields in $\mathbb{R}^3$. Forms corresponding to work, flux, and mass.
19	6.5	Examples of work, flux, and mass calculations.
20	6.6	"Piece with boundary" of a manifold, boundary orientation.
21	6.7	Exterior derivative d of a k-form. $d^2 = 0$, product rule.
22	6.8	Dictionary between d and gradient, curl, and divergence in $\mathbb{R}^3$.
23	6.10	Stokes' Theorem (informal proof only).
24	6.11	Stokes' Theorem in $\mathbb{R}^3$; standard integral theorems of vector calculus.
25	6.11	Integral theorems of vector calculus: examples.
26	6.13	Potentials, conservative vector fields, Poincaré Lemma

Notes:

1. This syllabus is designed for a 1-quarter course with 30 academic hours of instruction. It is sectioned into 26 lectures; this leaves 2 lectures available for in-class midterm exams and 2 lectures for review (or holidays). It is based on the following textbook:

   - *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fourth edition* by John H. Hubbard and Barbara Burke Hubbard.

2. The Math 31H Honors Calculus sequence is a rigorous treatment of multivariable calculus, including linear algebra and differential forms, for a self-selected population of students who have scored a 5 on the Advanced Placement Calculus BC exam. Math 31AH, 31BH, and 31CH substitute respectively for the standard calculus courses 20F (soon to be 18), 20C, and 20E; students who complete the sequence are also exempt from Math 109 due to the emphasis on proof. A minimum grade of B- in each course is required to continue in the sequence. The textbook includes more material than can be covered in three quarters, so it is necessary to be selective, especially about which of the major theorems can be fully proved in class. Scheduling midterm exams outside of class is an option for securing more time for course material. The Honors sequence is more rigorous and theoretically-oriented than the standard calculus sequence, but students should still learn to compute as well as to prove.

   Math 31CH covers the integral calculus of multivariable functions, in the general setting of $\mathbb{R}^n$. Main topics are determinants in $\mathbb{R}^n$, volumes of manifolds in $\mathbb{R}^n$, integration of differential forms, and Stokes' Theorem.

3. The treatment of vector calculus in this course is in the general setting of $\mathbb{R}^n$, in contrast to Math 20E which is restricted to $n = 2$ or 3. Although 31CH students will have a deeper understanding of the concepts, they may experience a language or notational barrier when taking subsequent applied courses such as engineering or physics. Instructors should address the issue of translation across this barrier, for example stressing the dictionary between differential forms and vector fields.

4. This syllabus optimistically assumes that Section 4.1, defining the integral in $\mathbb{R}^n$, was covered in 31BH. If not, the first two lectures of 31CH should be devoted to it. In that case the discussion of Stokes' Theorem may need to be shortened and Section 6.12 omitted.

5. There is no time to cover Sections 4.2, 4.3, and 4.4.

6. Note that the "spherical" coordinates used in Section 4.10 and elsewhere are not the standard ones, but more like latitude and longitude.

7. If time permits, it is nice to say something about how symmetry arguments apply to integrals in $\mathbb{R}^n$. For example, if the domain of integration is a union $A \cup B$, where A and B are related by an isometry, then the change of variables given by the isometry relates the integrals over the two subsets.