Mathematics 31AH: "Honors Linear Algebra" Syllabus (revised September 2016)

Lecture schedule based on:
Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fifth edition by John H. Hubbard and Barbara Burke Hubbard.

Lecture	Section(s)	Topic(s)
0	0.1	Advice on reading mathematics; students should read this on their own.
1	0.2, 0.3	Mathematical notation and language. Quantifiers, negation, set theory. Note the convention in this text that 0 is a natural number.
2	0.4	Functions. Domain, codomain, image, one-to-one and onto, composition, inverse.
3	1.1	Points and vectors (not the same!) in $\mathbb{R}^n$, vector operations, subspaces.
4	1.1, 1.2	Standard basis, matrices, matrix operations.
5	1.2	Inverse matrix, transpose.
6	1.3	Linear transformations, matrix representation of a linear transformation.
7	1.3	Geometric examples of linear transformations. Correspondence between operations on linear transformations and on their matrices.
8	1.4	Geometry of $\mathbb{R}^n$. Dot product, length, angle, Schwarz and triangle inequalities.
9	1.4	Matrix "length" (norm). Determinants in $\mathbb{R}^2$ and $\mathbb{R}^3$. Cross product.
10	1.4, 2.1	Determinants and volume. Row operations, echelon form.
11	2.2	Solving linear systems by row reduction. Pivotal variables.
12	2.2	Existence and uniqueness of solutions to linear systems. Geometric interpretation via intersections of (hyper)planes.
13	2.3	Computing matrix inverses. Elementary matrices.
14	2.3	Proof of the matrix inversion algorithm.
15	2.4	Linear combinations, span, linear independence. Number of spanning or independent vectors in $\mathbb{R}^n$.
16	2.4	Basis, orthonormal basis. Dimension Theorem 2.4.21.
17	2.5	Kernel and image of a linear transformation.
18	2.5	Finding bases for kernel and image. Rank.
19	2.5	Dimension formula (Theorem 2.5.8). Equality of row rank and column rank.
20	2.5	Examples/applications (optional): Polynomial interpolation, partial fractions.
21	***	Orthogonal projection onto a subspace (not in the textbook).
22	***	Least squares "solution" of an inconsistent linear system (not in the textbook).
23	2.7	Eigenvalues and eigenvectors of a square matrix. Diagonalization.
24	2.7	Linear independence of eigenvectors belonging to distinct eigenvalues.
25	2.7	Computing eigenvalues and eigenvectors. Note that the "standard" method via the characteristic polynomial is not introduced until much later (Section 4.8, Math 31CH).
26	2.6	Abstract vector spaces (optional). Alternatively, begin section 0.5 on the real number system.

Notes:

1. This syllabus is designed for a 1-quarter course with 30 academic hours of instruction. It is sectioned into 26 lectures; this leaves 2 lectures available for in-class midterm exams and 2 lectures for review (or holidays). It is based on the following textbook:

   - *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fourth edition* by John H. Hubbard and Barbara Burke Hubbard.

2. The Math 31H Honors Calculus sequence is a rigorous treatment of multivariable calculus, including linear algebra and differential forms, for a self-selected population of students who have scored a 5 on the Advanced Placement Calculus BC exam. Math 31AH, 31BH, and 31CH substitute respectively for the standard calculus courses 20F (soon to be 18), 20C, and 20E; students who complete the sequence are also exempt from Math 109 due to the emphasis on proof. A minimum grade of B- in each course is required to continue in the sequence. The textbook includes more material than can be covered in three quarters, so it is necessary to be selective, especially about which of the major theorems can be fully proved in class. Scheduling midterm exams outside of class is an option for securing more time for course material. The Honors sequence is more rigorous and theoretically-oriented than the standard calculus sequence, but students should still learn to compute as well as to prove.

   Math 31AH is a thorough treatment of linear algebra in $\mathbb{R}^n$, providing the foundation for the rest of the sequence to do multivariable calculus in the full generality of $\mathbb{R}^n$ as well. It begins with foundational material about sets, functions, and (optionally) the real number system, which can be used to introduce proof techniques and style.

3. Section 0.5 of the textbook covers the structure of the real number system including completeness and limits. This material can be covered in either 31AH or 31BH. It has more continuity (so to speak) with the 31BH calculus content, but in 31AH it can help to build students' familiarity with definitions, reasoning, and proof techniques. In these syllabi it has been included in 31BH.

4. Lectures 21 and 22 cover material that is not in the textbook. Students should be able to orthogonally project a vector into a subspace of $\mathbb{R}^n$, and construct an orthonormal basis for such a subspace. They should understand the least-squares technique of "solving" an inconsistent linear system $Ax = b$ by replacing it with the normal equations $A^TAx = A^Tb$. Abstract properties of projection operators can also be discussed.