

Mathematics 20C Syllabus (revised June 2021)

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Based on *Calculus: Early Transcendentals* (4th edition) by Jon Rogawski, Colin Adams, and Robert Franzosa. (See the next pages for explanatory comments.)

Lecture	Section	Topic
1	12.1	Vectors in the Plane
2	12.2	Three-Dimensional Space: Surfaces, Vectors, and Curves
3	12.3	Dot Product and the Angle Between Two Vectors
4	12.4	The Cross Product
5	12.5	Planes in 3-Space
6	13.1	Vector-Valued Functions
7–8	13.2	Calculus of Vector-Valued Functions
9	13.3	Arc Length and Speed
10	13.5	Motion in 3-Space
11	14.1	Functions of Two or More Variables
12	14.2	Limits and Continuity in Several Variables
13	14.3	Partial Derivatives
14	14.4	Differentiability, Tangent Planes, and Linear Approximation
15–16	14.5	The Gradient and Directional Derivatives
17	14.6	Multivariable Calculus Chain Rules
18–19	14.7	Optimization in Several Variables
20	14.8	Lagrange Multipliers: Optimizing with a Constraint
21–22	15.1	Integration in Two Variables
23–24	15.2	Double Integrals over More General Regions
25	15.3	Triple Integrals
26*	12.7	Cylindrical and Spherical Coordinates
	15.4	Integration in Polar, Cylindrical, and Spherical Coordinates

*These topics may be omitted for time.

Math 20C is the third quarter course in calculus for students majoring in Mathematics, Engineering and the sciences. Most students taking Math 20C will be continuing from Math 20B, but some new freshmen are placed directly into Math 20C with Advanced Placement credit. This makes teaching Math 20C in a Fall quarter interesting: it will have both incoming freshmen with Advanced Placement credit and continuing sophomores who failed (or did not take) Math 20C the previous Spring quarter.

Math 20C introduces vectors and three-dimensional geometry and covers multivariable differential calculus with an introduction to multiple integrals. Experience has shown that students have more trouble visualizing the geometry of space and understanding the geometrical significance of the calculus than they do with the actual computations. Thus, more emphasis should be placed on *what* is being computed and *why* it is being computed than on how to compute it.

The following syllabus uses 26 lectures of the 28 to 30 lectures available in a typical quarter. Some topics can be reduced/expanded as time requires/permits.

Remarks about Topics

Lec. 1. Sec. 12.1: Vectors in two dimensions.

Lec. 2. Sec. 12.2: Vectors in three dimensions. Also includes some basic surfaces.

Lec. 3. Sec. 12.3: The dot product. Include orthogonal projections.

Lec. 4. Sec. 12.4: The cross product. There is not enough time for a full treatment of determinants, so show how to compute three-by-three determinants, but do not go through all of the properties. Focus instead on computing cross products.

Lec. 5. Sec. 12.5: Equations of planes in three dimensions.

Lec. 6. Sec. 13.1: Vector-valued functions.

Lec. 7–8. Sec. 13.2: Calculus of vector-valued functions. Emphasize that calculus of vector-valued functions is calculus of real-valued functions in several components.

Lec. 9. Sec. 13.3: Arc length and speed. Introduce arc length as the integral of speed. (Arc length is not generally treated in Math 20A or Math 20B since parameterizations are not discussed until Math 20C.)

Lec. 10. Sec. 13.5: Motion in three dimensions.

Lec. 11. Sec. 14.1: Real-valued functions of more than one variable. We usually focus on functions of two variables.

Lec. 12. Sec. 14.2: Limits and continuity in several variables. This discussion should be kept informal. The epsilon-delta definition is not covered in Math 20A, and so is not necessary here. Aim for intuitive understanding.

Lec. 13. Sec. 14.3: Partial derivatives of a real valued function.

Lec. 14. Sec. 14.4: Differentiability, tangent planes, and linear approximation. Students should

understand the connection between tangent planes and linear approximation and that differentiability is more than just existence of partial derivatives.

Lec. 15–16. Sec. 14.5: Gradients and directional derivatives. Students should understand the geometric significance of the gradient and not just the formal computational definition.

Lec. 17. Sec. 14.6: This section is called “Multivariable Calculus Chain Rules” (plural) because this text does not give the most general version of the chain rule in terms of the total derivative. Rather than giving general formulas with n variables, focus on examples involving compositions of functions of two or three variables.

Lec. 18–19. Sec. 3.3: Optimization in several variables. Attention should be confined to real-valued functions of two variables.

Lec. 20. Sec. 3.4: The Method of Lagrange multipliers. The subsection “Lagrange Multipliers with Multiple Constraints” is optional.

Lec. 21–22. Sec. 15.1: The double integral over a rectangular region. There is a lot included in this section: The double integral as a volume, the double integral as a limit of Riemann sums, integration theorems, iterated integrals (for two variables), changing the order of integration. Stress that the equality of double integrals with iterated integrals is a theorem (Fubini’s theorem) and not the definition

Lec. 23–24. Sec. 15.2: Double integrals over more general regions. Changing the order of integration.

Lec. 25. Sec. 15.3: Triple integrals. Emphasize that triple integrals are just the natural extension of double integrals.

Lec. 26. Sec. 12.7 & 15.4: Cylindrical and spherical coordinates are introduced in Section 12.7. Emphasize that polar and cylindrical coordinates are closely related. Section 15.4 discusses integration in polar, cylindrical, and spherical coordinates. This topic is covered in greater generality in Math 20E. For Math 20C, a brief introduction to integration in polar, cylindrical, and spherical coordinates is enough. Triple integrals in spherical coordinates may be skipped if time is short.