Mathematics 20B Syllabus (revised May 2021)

Based on *Calculus: Early Transcendentals* (4th edition) by Jon Rogawski, Colin Adams, and Robert Franzosa. (See the next pages for explanatory comments.)

Lecture	**Section**	**Topic**
1	5.2–5.5	Review of the Fundamental Theorem
2	5.6	Net Change as the Integral of a Rate of Change
3	5.7	The Substitution Method
4	6.1	Area Between Two Curves
	6.2	Setting Up Integrals: Volume, Density, Average Value
5	6.3	Volumes of Revolution: Disks and Washers
6	7.1	Integration by Parts
7	11.3	Polar Coordinates
8	11.4	Area and Arc Length in Polar Coordinates
9	S.1	Complex Numbers
10	S.2	Complex Exponentials
11	7.2	Trigonometric Integrals
	7.4	Integrals Involving Hyperbolic and Inverse Hyperbolic Functions
	S.3	Integration of Functions which Take Complex Values
12	7.3	Trigonometric Substitution
13–14	7.5	The Method of Partial Fractions
	S.4	The Fundamental Theorem of Algebra
	S.5	Partial Fraction Expansions (PFE)
15–16	7.7	Improper Integrals
17	10.1	Sequences
18	10.2	Summing an Infinite Series
19	10.3	Convergence of Series with Positive Terms
20	10.4	Absolute and Conditional Convergence
21	10.5	The Ratio and Root Tests and Strategies for Choosing Tests
22–23	10.6	Power Series
24–25	10.8	Taylor Series

Note: Tell students to download the Math 20B Supplement to Rogawski. This supplement adds material to Chapters 7 of Rogawski. The current version is at

`http://www.math.ucsd.edu/~syllabi/`

Sections from the Math 20B Supplement appear above as S.1, S.2, S.3, S.4, and S.5.

Math 20B is the second quarter calculus course for students majoring in mathematics, engineering and the sciences. Most students taking Math 20B in the Winter or Spring quarters will have just finished Math 20A during the previous quarter. However, the Math 20B students in the Fall quarter are a particularly diverse group: some are placed into 20B directly from high school with a high score on an AP exam, while others may have transferred from a community college or other university. With this in mind, it may be worthwhile to spend an extra lecture to review antiderivatives when teaching Math 20B in the Fall. A couple of sections may be omitted to make time for review, or if time is short for other reasons.

Solid intuitive understanding of the concepts should be preferred to rigorous proof in this course. At this level, deeper understanding may result from a graphical argument emphasizing the conceptual issues more than from a proof based on unfamiliar technical manipulation or unstated axioms about the real number system.

It should perhaps be specifically noted that the discussion of sequences and series should avoid technical details by referring to known results for functions of a real variable: there is only time for a brief introductory treatment. The ultimate goal is to get students to appreciate that power series can be used to represent functions (something of particular importance to physics and engineering students).

Remarks about Topics

Lec. 1. Sec. 5.2-5.5: Review of the Fundamental Theorem of Calculus

Lec. 2. Sec. 5.6: Total change as the integral of a rate

Lec. 3. Sec. 5.7: The substitution method for integrals

Lec. 4. Sec. 6.1–6.2: Areas between curves, computing volumes using the "slicing" technique (using pieces of known cross-sectional area), average values of a function. *Skip the discussions of density and flow rate in Section 6.2.*

Lec. 5. Sec. 6.3: Computing the volume of solids of revolution as a special case of Sec. 6.2.

Lec. 6. Sec. 7.1: Integration by parts

Lec. 7–8. Sec. 11.3-11.4: Polar coordinates; areas in polar coordinates. *Skip the discussion of arc length at the end of Section 11.4.* (That discussion is usually left for multivariable calculus.) The use of the cosine double-angle identity to integrate functions involving sine squared and cosine squared will have to be introduced here.

Lec. 9–10. Supp. 1–2: Complex numbers and complex exponentials: Discuss de Moivre's theorem, complex roots, and Euler's formula. Discuss how the trigonometric functions are related to the complex exponential and how trigonometric identities may be derived using the complex exponential.

Lec. 11. Sec. 7.2, 7.4, Supp. 3: Trigonometric integrals, integrals of hyperbolic functions: Illustrate that trigonometric integrals can be done using complex exponentials, with the goal of giving the students experience with complex exponentials (which is particularly useful to engineering students). Integrals of hyperbolic functions use real exponentials and can be done quickly. *Integrals of inverse hyperbolic functions should be skipped.*

Lec. 12. Sec. 7.3: Trigonometric substitution (If behind schedule, this topic may be omitted.)

Lec. 13–14. Supp. 4–5, Sec. 7.5: The fundamental theorem of algebra; partial fractions and integration of rational functions using partial fractions. The discussion in Section 4 of the Supplement introduces students to the complex viewpoint of partial fraction expansions (PFE). Keep in mind that if a PFE is obtained using complex numbers, no complex numbers can appear in denominators when computing antiderivatives. (If Section 7.3 was skipped, be sure to use the Method of Substitution to compute antiderivatives of $1/(x^2 + a^2)$.)

Lec. 15–16. Sec. 7.7: Improper integrals: Take care to emphasize that improper integrals are limits; this will be reinforced during the discussion of sequences and series. The comparison test will also show up again during the discussion of sequences and series.

Lec. 17. Sec. 10.1: Sequences: limits, convergence, and divergence. Emphasize that a sequence is a function of the positive integers. Refer students to Section 2.3 of Rogawski for the corresponding properties of limits for functions of a real variable, and Section 2.7 for definition of limit at infinity: there is not time to formally list the properties of limits in class. *The discussion of bounded and monotonic sequences may be omitted.*

Lec. 18. Sec. 10.2: Series: Use the geometric series and harmonic (or other) series to illustrate a convergent versus a divergent series. Illustrate with further examples.

Lec. 19. Sec. 10.3: Series with positive terms: the integral and comparison tests. Emphasize that these are just two variations of the idea of comparison.

Lec. 20. Sec. 10.4: Absolute convergence; the ratio and root tests; conditional convergence and the Leibnitz test for alternating series.

Lec. 21. Sec. 10.5: Emphasize that the ratio and root tests are based on comparison with an appropriate geometric series, but avoid the technical proofs.

Lec. 22–23. Sec. 10.6: Power series: Emphasize that power series are important because they are so much like polynomials. Note that the radius of convergence of a power series can usually be checked using the ratio test (or root test).

Lec. 24–25. Sec. 10.8: Taylor series: Emphasize that the process for finding coefficients of a Taylor series for a function comes from treating the function as a polynomial. State the remainder theorem, but avoid discussing the proof.