

Mathematics 20A Syllabus (revised May 2021)

Based on *Calculus: Early Transcendentals* (4th edition) by Jon Rogawski, Colin Adams, and Robert Franzosa. (See the next pages for explanatory comments.)

Lecture	Section	Topic
0	1.4	Trigonometric Functions
	1.5	Inverse Functions
	1.6	Exponential and Logarithmic Functions
	1.7	Technology: Calculators and Computers
1*	2.1	The Limit Idea: Instantaneous Velocity and Tangent Lines
	2.2	Investigating Limits
2	2.3	Basic Limit Laws
	2.4	Limits and Continuity
3	2.5	Indeterminate Forms
	2.7	Limits at Infinity
4	2.6	The Squeeze Theorem and Trigonometric Limits
	2.8	Intermediate Value Theorem
5	3.1	Definition of the Derivative
6	3.2	The Derivative as a Function
7	3.3	Product and Quotient Rules
8	3.4	Rates of Change
	3.5	Higher Derivatives
9	3.6	Trigonometric Functions
10	3.7	The Chain Rule
11	3.8	Implicit Differentiation
12	3.9	Derivatives of General Exponential and Logarithmic Functions
13	3.10	Related Rates
14	4.1	Linear Approximation and Applications
15	4.2	Extreme Values
16	4.3	The Mean Value Theorem and Monotonicity
17	4.4	The Second Derivative and Concavity
18	4.5	L'Hôpital's Rule
19	4.6	Analyzing and Sketching Graphs of Functions
20	4.7	Applied Optimization
21	5.1	Approximating and Computing Area
22	5.2	The Definite Integral
23	5.3	The Indefinite Integral
24	5.4	The Fundamental Theorem of Calculus, Part I
25	5.5	The Fundamental Theorem of Calculus, Part II

*The epsilon-delta definition, from Section 2.9, is not covered in this course.

Math 20A is the first quarter calculus course for students majoring in Mathematics, Engineering and the sciences. The vast majority of students taking Math 20A have had a year of high school calculus, but did not score well enough on the Advanced Placement examination to start with Math 20B or Math 20C. They may be familiar with the mechanics of solving calculus problems, but weak on conceptual understanding, which should be emphasized in Math 20A.

Students should be encouraged to use a graphing calculator (the TI-86 is standard) or a computer to facilitate computation and graphing, and build—not substitute for—understanding. Calculators with the capability to perform symbolic manipulation, such as the TI-89 or TI-92, are generally disallowed on exams; in fact, many instructors prefer not to allow calculators of any type on exams. Students should be clearly informed which calculators are permissible and, in particular, which of their capabilities (if any) may be used on exams.

Solid intuitive understanding of the concepts should be preferred to rigorous proof in this course. At this level, deeper understanding may result from a graphical argument emphasizing the conceptual issues more than from a proof based on unfamiliar technical manipulation or unstated axioms about the real number system. The following generic syllabus takes 25 lectures (out of the 28 to 30 available in a typical quarter). It is intended to be an approximation: in practice, some topics may take less time than allotted, while other topics may take more time. Supplementary or review lectures may be added if time permits.

Remarks about Topics

Sec. 1.1, Sec. 1.2 and Sec. 1.3. These sections are considered review and are generally not discussed in class. Students should read and should be assigned exercises from these sections. Students should already be familiar with this material, but may benefit from being reminded about it with some assigned exercises.

Lec. 0. Sec. 1.4, 1.5, 1.6: These sections are also considered review. However, it is sometimes useful to review concepts from these sections during class, when appropriate, time permitting. Assigning homework exercises from these sections is recommended. Consider asking your TAs to discuss the exercises from these sections during the first discussion meeting.

Sec. 1.4: Trigonometric Functions: The key concept from this section is the definition of the trigonometric functions on the unit circle. The addition (and double-angle) identities become important later on, chiefly when integrating in Math 20B.

Sec. 1.5: Inverse Functions: Emphasis in this section should be placed on the inverse trigonometric functions, which provide examples of restricting the domain of a function to obtain an invertible function.

Sec. 1.6: Exponential and Logarithmic Functions: A full development of the natural exponential requires calculus, but the students are expected to understand that the natural exponential and the natural logarithm are inverses of one another. They should also know the definition of the hyperbolic functions.

Sec. 1.7: Technology: Calculators and Computers. Students should read this section on their own. The realization that calculator-generated graphs can be unreliable in various ways can motivate calculus as a way to investigate and justify the correctness of such graphs.

Lec. 1. Sec. 2.1 & 2.2: These sections, which introduce the idea of limits, are usually done together. The ideas behind rates of change and tangent lines are used to motivate the notion of a limit, which are explored using numerical and graphical approaches. It should be emphasized that numerical estimation is not the same as actually computing a limit analytically.

Note: The ϵ - δ definition of a limit (in Section 2.9 of this text) is not discussed in Math 20A.

Lec. 2. Sec. 2.3 & 2.4: These two sections (Basic Limit Laws & Limits and Continuity) are usually treated together.

Lec. 3. Sec. 2.5 & 2.7: These two topics (Indeterminate Forms & Limits at Infinity) are usually done together with limits at infinity considered as a variation of finite limits. (As a reminder, the formal definition of limits is usually not given in this class.)

Lec. 4. Sec. 2.6 & 2.8: *The Squeeze Theorem and Trigonometric Limits:* The trigonometric limits should be done carefully, since they will be used later to obtain the trigonometric differentiation formulas. *Intermediate Value Theorem:* Illustrate the theorem with examples rather than provide a proof.

Lec. 5. Sec. 3.1: The limit definition of the derivative at a point is given. No general derivative formulas are given in this section. (They start to appear in the next section.)

Lec. 6. Sec. 3.2: The derivative function is defined. This includes a discussion of the derivative formulas for polynomials and exponential functions. Most students will be familiar with the differentiation rules from high school, but may not understand that these are consequences of the definition, not the definition itself.

Lec. 7. Sec. 3.3: The Product and Quotient Rules are introduced. Emphasis should be given on examples rather than rigorous proofs.

Lec. 8. Sec. 3.4 & 3.5: *Rates of Change:* Present one or two simple examples/applications of rates of change: ultimately, students should be able to recognize rates of change in many contexts. *Higher Derivatives:* This is usually presented with Sec. 3.4, as motion, velocity, and acceleration are a natural overlap between the two sections.

Lec. 9. Sec. 3.6: Emphasize how the formula for the derivatives of the sine and cosine follow naturally from the definition of the derivative.

Lec. 10. Sec. 3.7: Emphasize that the Chain Rule applies to composite functions. A brief reminder about composition may be in order. A rigorous proof is not necessary.

Lec. 11. Sec. 3.8: Emphasize the relationship between implicit differentiation and the Chain Rule. Derivatives of inverse functions are treated as an example of implicit differentiation. No explicit formula is given for the derivative of an inverse function. (This is a change from the 2nd edition of the textbook, where it was treated in its own section.)

Lec. 12. Sec. 3.9: The derivative is obtained for general exponential functions (using the Chain Rule) and general logarithmic functions (using implicit differentiation). Passing familiarity with the hyperbolic and inverse hyperbolic functions is sufficient.

Lec. 13. Sec. 3.10: Related rates are hard for students since it combines many of the non-formulaic aspects of calculus: the meaning of the derivative and of the Chain Rule, functions which are not explicitly given by formulas and the use of a symbol such as x to represent both a function

and its value at a point.

Lec. 14. Sec. 4.1: Mention that linear approximation can be expressed using the language of differentials.

Lec. 15. Sec. 4.2: Extreme values. You can skip Rolle's Theorem.

Lec. 16. Sec. 4.3: Emphasize the geometric connection between the derivative and the monotonicity (increasing/decreasing) of a function and how this leads to the First Derivative Test. The Mean Value Theorem should be mentioned only in passing, since it is hard for students to appreciate its role at this level because it seems less obvious than many of the results it is used to prove.

Lec. 17. Sec. 4.4: Emphasize the geometric basis of the tests for concavity and inflection points of graphs and the second derivative test for extreme values.

Lec. 18. Sec. 4.5: L'Hôpital's Rule appears here because it may be needed to compute asymptotes in the next section.

Lec. 19. Sec. 4.6: Here the students use all the tools they have learned previously to characterize the behavior of graphs.

Lec. 20. Sec. 4.7: Students often find it difficult to translate an optimization problem into mathematical language: it is this process that should be carefully exhibited to them.

Lec. 21. Sec. 5.1: Sigma (summation) notation should be introduced.

Lec. 22. Sec. 5.2: The definition of the definite integral given in this section is a general one using norms of partitions. It is sufficient to consider the uniform partitions used in constructing the left-endpoint and right-endpoint approximations.

Lec. 23. Sec. 5.3: This section contains the basic antiderivatives. This section also contains a brief introduction to differential equations and initial value problems. These ideas are revisited in Section 5.5 in the context of the Fundamental Theorem of Calculus, Part II.

Lec. 24. Sec. 5.4: The Fundamental Theorem of Calculus, Part I: This topic is fundamental.

Lec. 25. Sec. 5.5: The Fundamental Theorem of Calculus, Part II: Emphasize that Part II of the fundamental theorem is a statement about the *existence* of antiderivatives.