

Math 184, Spring 2023, Midterm 2 practice problems

Here are some practice problems for Midterm 2. They are a mixture of problems taken from Fall 2019 and Winter 2022, so this list is not balanced around how long it takes to finish since it doesn't come from a single exam.

- (1) Let α, β, γ be constants and d a positive integer. Compute the coefficient of x^3 in the following formal power series:

$$\frac{(\alpha + \beta x)^d}{(1 - \gamma x)^4}.$$

- (2) For a positive integer n , let a_n be the number of partitions of n using only the parts 1 and 3. As usual, set $a_0 = 1$.
- (a) Show that $a_n = a_{n-3} + 1$ for all $n \geq 3$.
- (b) Find a *homogeneous* linear recurrence relation for a_n (make sure to state for which n it is valid) and give enough initial values so that your answer determines the whole sequence.
- (3) How many partitions of $[8]$ are there such that every block has the same size?
- (4) What is the coefficient of $x^3 y^2 z^5$ in $(x + y + z)^{10}$?
- (5) Express $\sum_{n \geq 0} (n - 4^n) x^n$ as a rational function.
- (6) What is the coefficient of x^{16} in $\sqrt{1 + 5x^2}$ when expanded as a formal power series?
- (7) Let a_n be the sequence defined by the recurrence

$$a_n = 6a_{n-1} - 9a_{n-2} \quad \text{for } n \geq 2, \quad a_0 = a_1 = 1.$$

Find a closed formula for a_n .

- (8) Let $n \geq 3$ be an integer. Evaluate $\sum_{\substack{0 \leq i \leq n \\ i \text{ even}}} i(i-1) \binom{n}{i}$.