Math 184, Spring 2023, Final exam study guide

- The final exam will take place in the usual lecture hall, Center 216, on **Tuesday**, June 13 from 8AM–11AM (in the morning...).
- This is a closed-book, closed-note exam. Just bring something to write with. *I will provide a brief formula sheet, see the last page for what it looks like.*
- Bring your ID.

1. Content

The final exam is cumulative (i.e., covers everything in the notes and Homeworks 1–7). There will be more emphasis on material that was not on the midterms, i.e., material from Homeworks 5–7.

The old topics (taken from previous study guides):

- Methods to deal with bijections (sum principle, subtraction principle, product principle, and actually finding bijections)
- Falling factorials
- Permutations (and variations)
- Words and the problems they apply to
- Choice problems: subsets, multisets, Poker hands
- (Weak) compositions and the problems they apply to
- Set partitions: Stirling numbers, relation to falling factorials, methods for directly counting, recursive formula and related bijections
- Integer partitions: definitions, some basic bijections
- Binomial and multinomial theorems: statement, how to simplify sums
- Formal power series: algebra of FPS (addition, multiplication, derivatives, composition), extracting coefficients of terms, general binomial theorem
- Ordinary generating functions: finding simple expressions for generating function given a sequence (either as an explicit formula or as recurrence)
- Linear recurrence relations: using characteristic polynomial to find closed formula, solving some basic inhomogeneous recurrence relations

Topics since Midterm 2:

- Partition generating functions (product formulas, getting recurrences, proving identities)
- Catalan numbers and solving quadratic equations
- Exponential generating functions:
  - structures, products, and exponential formula.
  - Key examples: permutations, set partitions, labeled graphs.
  - Using derivatives to extract recurrence relations.
- Lagrange inversion formula (Cayley's formula n^{n-2} isn't specifically important, but rather the process behind it is)
- Inclusion-exclusion
- Möbius inversion

You can review the podcasted lectures through the "media gallery" in Canvas. Unfortunately, there are 2 problematic videos:

- The camera was not working for Lecture 3. For that lecture, you can reference the two videos from Winter 2022 posted there. The relevant portions are Jan6: 48:30-end and Jan11: 00:00-20:00
- The audio was recorded at a very low volume for Lecture 6. It is possible to hear the dialogue, but an alternative is to look at the relevant portions of the Jan18 and Jan20 videos posted under Winter 2022.

2. Practice problems

The best way to study is to solve a wide variety of problems. First, I recommend reviewing all of your homework assignments and both midterms. Solutions for all of them can be found in the Discord server (look in the pinned message for each assignment's channel). Beyond that, I'll give two more sources of new problems: old exams and exercises from Bóna.

Here is a collection of past final exam problems that I've given in Math 184 (including the complete Fall 2019 exam):

https://mathweb.ucsd.edu/~ssam/184/practice-final.pdf

3. Extra practice from Bóna

If you want additional practice with the material, I've listed some relevant exercises from Bóna. Everything is in reference to the **4th edition** of the textbook. I repeat the content from the Midterm 1 and 2 study guides for your convenience.

3.1. **Midterm 1.** All of these have solutions in the book

- Chapter 3: 1-3, 6-12, 14-15, 18-20, 23

The following do not have solutions in the book. I will not provide a solutions manual due to time constraints. However, I am happy to discuss these problems either in office hours or over Discord.

- Chapter 3: 27-37, 41-42, 44-47, 49-50, 52-55
- Chapter 5: 18, 21, 23-26, 36-37

3.2. **Midterm 2.** All of these have solutions in the book:

- Chapter 4: 3-4, 18-19, 26-28
- Chapter 5: 6-7, 11
- Chapter 8: 1-2, 4-5

The following do not have solutions in the book:

- Chapter 4: 41, 43-47, 50
- Chapter 5: 20, 22
- Chapter 8: 25-28, 38-40

3.3. **Post Midterm 2.** Problems with solutions in the book:

- Chapter 7: 3–5, 7–8
- Chapter 8: 8, 11, 21–22

Problems without solutions in the book:

- Chapter 7: 17–20, 28–31
- Chapter 8: 47–48

Formula sheet

Comments: This sheet will be provided for you along with the final exam. Note that not every formula we discuss in lecture is necessary for the exam. In trying to distinguish, be mindful of whether something is important for solving homework problems or is just an auxiliary fact which was derived simply to illustrate a technique. This sheet provides some formulas which I think are easy to misremember.

- The number of k-element multisets of an n-element set is $\binom{n+k-1}{k}$.
- The number of weak compositions of n with k parts is $\binom{n+k-1}{k-1}$.
- The change of basis between powers and falling factorials is

$$x^n = \sum_{k=0}^{n} S(n,k)(x)_k$$

  where $S(n,k)$ is the Stirling number.
- If d, n are non-negative integers, then

$$\binom{-d}{n} = (-1)^n \binom{d+n-1}{n}.$$

- Lagrange inversion formula: if $G(x)$ is a formal power series with nonzero constant term, then there is a unique formal power series $A(x)$ such that

$$A(x) = xG(A(x)).$$

  Furthermore, $A(x)$ has no constant term and for $n \geq 1$, we have

$$[x^n]A(x) = \frac{1}{n}[x^{n-1}](G(x)^n).$$

- Given an alphabet of size k, the number of words of period d is

$$\omega(d) = \sum_{e|d} \mu(d/e)k^e,$$

  where μ is the Möbius function, and the number of necklaces of length n is

$$\sum_{d|n} \frac{\omega(d)}{d}.$$