Math 184, Winter 2022
Homework 6
Due: Friday, Mar. 4 by 11:59PM via Gradescope (late homework will not be accepted)

Explanations should be given for your solutions. Use complete sentences.

(1) For $n > 0$, let h_n be the number of bijections $f\colon [n] \to [n]$ with the property that $f \circ f \circ f$ is the identity function. We set $h_0 = 1$.
   (a) Give a simple expression for the exponential generating function $H(x) = \sum_{n \geq 0} \frac{h_n}{n!} x^n$.
   (b) Use your answer in (a) to get a recurrence for h_n.

(2) In Example 7.14, we counted the number of labeled trees on 3 and 4 vertices by considering all types of unlabeled trees and counting how many different labelings each one has. Do the same for 5 and 6 vertices.

   To make it easier not to go down the wrong path: there are 3 types of unlabeled trees on 5 vertices and 6 types of unlabeled trees on 6 vertices. Also your answers should add up to $5^3 = 125$ and $6^4 = 1296$, respectively.

(3) How many ways are there to list the letters of ORANGECHICKEN so that no two consecutive letters are the same?

(4) We have $n > 1$ married couples ($2n$ people in total).
   (a) How many ways can we have the $2n$ people stand in a line so that no person is standing next to their spouse?
   (b) How many ways can we have the $2n$ people stand in a circle so that no person is standing next to their spouse (any rotation of the circle is considered an equivalent arrangement)?

(5) How many necklaces are there of length n using k different colors for the beads where n is:
   (a) 8
   (b) 12
   (c) 30

1. Extra problems (don't turn in)

(6) How many positive integers ≤ 1000 are neither perfect squares nor perfect cubes? [Recall that a perfect square is an integer of the form n^2 where n is an integer, and a perfect cube is an integer of the form n^3 where n is an integer.]