Math 190A, Fall 2022
Study guide for final exam

- The exam is in-class on December 9, 11:30AM–2:30PM.
- There are no notes or books allowed.
- I will provide paper, so do not bring a blue book.
- Content: everything, the final exam is cumulative. However, expect more emphasis on new material.

The format will be similar to the midterms, just longer (in terms of scaling, if the midterms were "5 problems" then the final will be somewhere in the range of 7–8 problems depending on their complexity). Here is what I expect you to know:

- All items from study guides 1 and 2. In principle, a question or topic from either midterm could reappear.
- All homework problems from Homeworks 5, 6, and 7 (ignore optional problems) whether they were graded or not. You might see them in the exact same form, either partially or in whole. You might also see them with slight modifications.
- Definitions: you may be asked to repeat definitions. You don't need to memorize the exact wording, but you do need to give something that has the exact same meaning. To make this simple for you and me, just stick with our definition and don't use results (for example, for "closed subset" our definition is "complement is open"; one could also say "contains all of its limit points" which is technically correct, but for us that is a result, and not the original definition). Here are the ones you could be asked:
  (1) compact
  (2) limit point compact
  (3) locally compact
  (4) compactification, one-point compactification
  (5) Alexandroff extension X^* (you would be asked to say which sets are open, but not need to prove that this is a topology unless explicitly asked)
  (6) second-countable
  (7) T_1-space, regular, normal
  (8) locally Euclidean of dimension n, n-manifold
  (9) partition of unity subordinate to a finite open covering
- Examples: this was not included before, but now that we have more tools, it makes sense. Namely, we discussed a few key examples of spaces and whether or not they have certain properties (connected, path-connected, compact, locally compact, Hausdorff, etc.). A good way to learn the standard techniques is to study the examples we discussed after introducing these concepts. When asked to prove something, you can use theorems without reproving them, but to make grading easier, state what theorem you're using (example: S^n is closed because ... and bounded because ... and therefore is compact by the Heine–Borel theorem).

  Here are the examples you should be comfortable with:
  - Euclidean space $\mathbf{R}^n$
  - spheres S^n
  - real and complex projective space $\mathbf{RP}^n$ and $\mathbf{CP}^n$
  - matrix groups $\mathbf{GL}_n(\mathbf{R})$, $\mathbf{O}_n(\mathbf{R})$, $\mathbf{U}(n)$ (important: we did not discuss whether or not they are connected so don't worry about that)

You're not responsible for the details of the topologist's sine curve or comb space. While there aren't many examples listed here, you could be asked about new examples that either use theorems below applied to them (for example, what can you say about $S^2 \times \mathbf{R}^4$?) or use the ideas used to study them.

- Statements of propositions / theorems: you may be asked to state a formula or to complete or fully give the statement of a theorem. Again, the exact wording is not necessary, but the meaning should be exactly the same. Anything not mentioned here you still want to know because it might be useful to prove something else.
  (1) Prop. 3.3.14 and its consequences
  (2) Thm 4.1.15 (Heine–Borel theorem)
  (3) Prop 4.3.12 and Cor 4.3.13: properties of X^*
  (4) Prop 4.3.19: equivalent formulation of locally compact for Hausdorff spaces
  (5) Prop 5.2.4, 5.2.5, 5.2.6: conditions that imply normal
  (6) Thm 5.3.1 Urysohn's lemma (the first draft of the notes did not include the condition that X has to be T_1 to start; make sure you check the latest version for the updated statement)
  (7) Thm 5.4.1 Urysohn metrization theorem
- Proofs of propositions / theorems: you may be asked to reprove a proposition or theorem from class, either in part or in whole. Some of these proofs rely on previous results. Of course you don't need to reprove those previous results, but you should make it clear that a previous result is being used. Here are the ones you want to know:
  (1) Prop 4.1.6: compact subspace of Hausdorff space is closed
  (2) Prop 4.1.18: continuous image of compact space is compact
  (3) Cor 4.1.21
  (4) Prop 4.1.24
  (5) Prop 4.2.2: compact implies limit point compact
  (6) Prop 4.3.8: uniqueness of one-point compactification
  (7) Prop 5.2.2
  (8) Prop 5.4.4: $\mathbf{R}^\omega$ is metrizable (the proof has many components, so I would not ask to prove from scratch; instead, I might give you some information, e.g., you're given the metric and asked to prove that the topologies agree)
- New problems: there will be 2 or 3 problems that are not covered by the above, but expect them to be around the average difficulty of the homework problems you've seen.