Math 188, Fall 2022
Homework 7

This is for practice for the final exam only, don't turn it in. Problem #4 deals with the short §7.4 which we likely won't cover until the last lecture. I don't plan to include §7.4 on the final exam; I'll leave it as a problem for interested students.

(1) Do the case of general n of Example 7.11, i.e., give a formula for the number of necklaces (considered equivalent up to reflection) of length n using an alphabet of size k.

(2) Consider assigning one of k colors to each of the entries of a 3×3 matrix.
  (a) How many ways are there to do this if we consider two colorings the same if they differ by rotation? To be explicit, one rotation clockwise means:
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \mapsto \begin{bmatrix} g & d & a \\ h & e & b \\ i & f & c \end{bmatrix}$$
  (b) How many colorings (up to rotation) are there that use exactly 3 different colors from the k, each used to color 3 entries?

(3) In Theorem 7.9, take $X = [n]$, $Y = [d]$, and $G = \mathfrak{S}_n$ with the natural action on X.
  (a) Find a bijection between G-orbits on Y^X and weak compositions; give a closed formula for their number using this interpretation.
  (b) By varying d, explain how the equality between the expression in Theorem 7.9 and your answer to (a) gives a new proof for Corollary 3.30.

1. Optional problems

(4) Let p be a prime and $n \geq p$. Use the method of §7.4 for the following:

(a) Show that
$$S(n,k) \equiv S(n-p,k-p) + S(n-p+1,k) \pmod{p}.$$

(b) Show that
$$c(n,k) \equiv c(n-p,k-p) - c(n-p,k-1) \pmod{p}.$$