Math 154, Winter 2019
Homework 2
Due: Tuesday, Jan. 22 by noon in basement of AP&M

(1) How many ways are there to list the letters of the word LAJOLLA?

(2) How many integers are there between 1000 and 9999 in which all digits are different?

(3) How many ways are there to pick 5 days in February (assume it's not a leap year) such that at most one Sunday is picked?

(4) (a) We want to select three subsets A, B, and C of $[n]$ so that $A \subseteq C$ and $B \subseteq C$. How many ways can this be done?
(b) We want to select three subsets A, B, and C of $[n]$ so that $A \subseteq C$, $B \subseteq C$, and $A \cap B \neq \emptyset$. How many ways can this be done?

(5) A "forward path" in the plane is a sequence of steps of the form $(1,0)$ and $(0,1)$.
(a) How many forward paths are there from $(0,0)$ to (a,b) where a,b are non-negative integers?
(b) How many forward paths are there from (a,b) to (c,d) where $c \geq a \geq 0$ and $d \geq b \geq 0$ are integers?
(c) Generalize this definition to d dimensions by only allowing steps which increase one of the coordinates by 1 (so $(1,0,0,\dots,0),(0,1,0,\dots,0),\dots,(0,0,0,\dots,1)$). How many forward paths are there from $(0,0,\dots,0)$ to $(a_1,a_2,\dots,a_d)$ where $a_1,\dots,a_d$ are non-negative integers?

(6) Let n and k be positive integers. Show that the number of ordered collections $(X_1,\dots,X_k)$, where each X_i is a subset of $[n]$, and $X_1 \cap X_2 \cap \cdots \cap X_k = \emptyset$ (i.e., there is no element which is in all of the X_i) is $(2^k-1)^n$.

For example, when $k=2$ and $n=2$, here are the 9 ordered collections:

$$\begin{array}{rrr} (\emptyset,\emptyset) & (\emptyset,\{1\}) & (\emptyset,\{2\}) \\ (\emptyset,\{1,2\}) & (\{1\},\emptyset) & (\{2\},\emptyset) \\ (\{1,2\},\emptyset) & (\{1\},\{2\}) & (\{2\},\{1\}). \end{array}$$