Math 154, Winter 2019
Homework 1
Due: Monday, Jan. 14 by 5PM in basement of AP&M

This homework is review of material that you have seen in previous courses. It is important that you can solve these problems since this course will use these skills.

(1) Prove that for all positive integers n,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

(2) Prove that every polynomial in x can be written as a linear combination of the polynomials
$$1, 2x - 1, (2x - 1)^2, (2x - 1)^3, (2x - 1)^4, \ldots.$$

(3) What are the eigenvalues of the following matrix? (This should not require any detailed computation.)
$$\begin{bmatrix} 1 & 5 & 0 & 4 & 0 \\ 0 & -1 & 4 & 6 & 1 \\ 0 & 0 & 1 & 6 & 2 \\ 0 & 0 & 0 & -8 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(4) Let A be a 5×5 matrix whose eigenvalues are $1, 3, -1, 0, \sqrt{2}$. What are the eigenvalues of $4A$? What about A^4?

(5) Find a formula for $\sum_{j=1}^{n}(3j + 5)$ and prove your answer.