

Math 184, Fall 2019

Homework 6

Due: **Monday**, December 2 by 3:00PM in homework box #2 in basement of AP&M

(late homework will not be accepted)

Explanations should be given for your solutions. Use complete sentences. Some hints are given on the next page.

- (1) (a) We have  $n$  distinguishable telephone polls. We want to paint each one either red, blue, green, or black such that an even number of them is red and an odd number of them is blue. How many ways can this be done?
- (b) Continuing with that situation, we add the colors white and yellow, but the total number of polls which are white or yellow must be even. How many ways are there to choose colors?

- (2) Let  $n$  be a positive integer. Given a group of  $n$  people, we want to divide them into nonempty committees and choose a chair and co-chair for each committee (they must be different people), and also choose one of the committees to be in charge of all of the others. Let  $h_n$  be the number of ways to do this and set  $h_0 = 1$ . Give a simple expression for the exponential generating function  $H(x) = \sum_{n \geq 0} \frac{h_n}{n!} x^n$ .

- (3) (a) Let  $k$  be a positive integer. Let  $a(k)_n$  be the number of set partitions of  $[n]$  into  $k$  blocks such that every block has at least 2 elements. Give a simple expression for the exponential generating function

$$A_k(x) = \sum_{n \geq 0} \frac{a(k)_n}{n!} x^n.$$

- (b) Let  $a_n$  be the number of set partitions of  $[n]$  such that every block has at least 2 elements. By convention,  $a_0 = 1$ . Give a simple expression for the exponential generating function

$$A(x) = \sum_{n \geq 0} \frac{a_n}{n!} x^n.$$

- (4) Let  $h_n$  be the number of bijections  $f: [n] \rightarrow [n]$  with the property that  $f \circ f \circ f$  is the identity function. Give a simple expression for the exponential generating function

$$H(x) = \sum_{n \geq 0} \frac{h_n}{n!} x^n.$$

- (5) For the following, write  $\exp(x)$  instead of  $e^x$  for ease of reading the superscript.

$F(x) = \sum_{n \geq 0} f_n x^n$  is a formal power series that satisfies the following identity:

$$F(x) = \exp\left(\frac{x}{2}(F(x) + 1)\right).$$

Find a formula for  $f_n$ .

Hints:

3a: This is like our derivation of the exponential generating function for Stirling numbers.

What happens for  $k = 1$ ?  $k = 2$ ?

5: Consider  $A(x) = x(F(x) + 1)$