Math 184A, Fall 2018
Homework 3
Due: Friday, Oct. 19 by 3:30PM in basement of AP&M

Some hints are given on the next page.

(1) Find the number of compositions of 56 into even pieces (i.e., compositions $(a_1, \dots, a_k)$ of 56 so that each a_i is even, and here k is allowed to vary).

(2) Find a simple formula for $S(n, n-2)$, i.e., the number of partitions of $[n]$ into $n-2$ blocks (assume that $n \geq 3$).

(3) Let $F(n)$ be the number of all partitions of $[n]$ such that every block has size ≥ 2. Prove that
$$B(n) = F(n) + F(n+1),$$
where $B(n)$ is the nth Bell number.

(4) In class, we showed that the number of compositions of n is 2^{n-1}. Find a bijection between the set of compositions of n and the set of subsets of $[n-1]$. In proving correctness of this bijection, you should not need to use the fact that these sets have size 2^{n-1}.

(5) Fix an integer $n \geq 2$. Call a composition $(a_1, \dots, a_k)$ of n **doubly even** if the number of a_i which are even is also even (i.e., there could be no even a_i, or 2 of them, or 4, or ...).

Show that the number of doubly even compositions of n is 2^{n-2}.

For example, if $n = 4$, then here are the 4 doubly even compositions of 4:
$$(2,2), \qquad (3,1), \qquad (1,3), \qquad (1,1,1,1).$$

Hints

(4) Partial sums

(5) Given a composition $\alpha = (a_1, \ldots, a_k)$, define another composition $\Phi(\alpha)$ by

$$\Phi(\alpha) = \begin{cases} (1, a_1 - 1, a_2, a_3, \ldots, a_k) & \text{if } a_1 > 1 \\ (a_2 + 1, a_3, \ldots, a_k) & \text{if } a_1 = 1 \end{cases}.$$

(in both cases, we didn't do anything to $a_3, \ldots, a_k$). Show that Φ defines a bijection between the set of doubly even compositions of n and the set of compositions of n which are not doubly even.