

Instructions for presentations:

## 1. SMITH NORMAL FORM

Things to definitely cover:

- Statement and examples of Smith normal form (SNF)
- Smith normal form gives classification of finitely generated abelian groups. Idea: every finitely generated abelian group is the cokernel of some integer matrix; now use SNF to describe the cokernel. Explain statement and proof.
- Submodules of free  $\mathbf{Z}$ -modules are still free. Explain statement and proof. Follows from classification: a free module can't have a torsion module as a submodule.

Optional (present if time permits):

- Generalizations to principal ideal domains (PID). State what is PID and why this is generalization.
- Computer demonstration. For example, Sage does it, see <http://www.williamstein.org/papers/ant/html/node21.html>

References:

- Wikipedia for an overview and summary [https://en.wikipedia.org/wiki/Smith\\_normal\\_form](https://en.wikipedia.org/wiki/Smith_normal_form)
- See §20.3 and §21.3 of <http://math.wisc.edu/~svs/490/SNF.pdf> for Smith normal form and classification of finitely generated abelian groups.  
(If you don't know what a principal ideal domain (PID) is, you can just substitute  $\mathbf{Z}$  in for  $R$  at all places.  $\mathbf{Z}$  is an example of a PID.)

## 2. UNIVERSAL COEFFICIENTS THEOREM

Things to definitely cover:

- Define tensor product of abelian groups and Tor for abelian groups. Give many examples.
- Statement of universal coefficients theorem.
- Some examples of universal coefficients theorem.

Optional (present if time permits):

- Proof of various parts of universal coefficients theorem
- Universal coefficient theorem for cohomology (see first part of Section 3.1 of Hatcher's book)

References:

- The general definition of tensor product of abelian groups can be found here: <http://isites.harvard.edu/fs/docs/icb.topic980968.files/tensor.pdf>  
You can also find a complete description of all possible cases for finitely generated abelian groups on p.179 of GIBLIN's book.
- Appendix A of Chapter 3 of Hatcher's book: <https://www.math.cornell.edu/~hatcher/AT/ATchapters.html> This is the best reference I can think of that minimizes the amount of additional information you need. This doesn't explain tensor product of abelian groups though.

### 3. ALGEBRAIC MORSE THEORY

Things to definitely cover:

- Definitions and main theorem in §11.3 of Kozlov's book. You may need some definitions earlier in Chapter 11. Look back as necessary, but start with §11.3.
- Example(s)

Optional (present if time permits):

- Version for CW complexes (and define CW complexes) in §11.2

References:

- Kozlov's book: <http://www.maths.ed.ac.uk/~aar/papers/kozlov.pdf>