Math 742, Spring 2016
Homework 6
Due: March 4

1. Exercises

(1) A ring R is **coherent** if every finitely generated ideal I is a finitely presented module.[1]
  (a) Suppose that R is coherent. Let $f\colon M \to N$ be a homomorphism of finitely presented modules. Show that $\ker f, \operatorname{coker} f, \operatorname{image} f$ are all finitely presented.
  (b) Let R be a noetherian ring. Show that the polynomial ring in infinitely many variables $R[x_1, x_2, \dots]$ is coherent.

(2) Show that if R is noetherian, then so is the power series ring $R[\![x_1, \dots, x_n]\!]$.[2]

(3) Let R be a noetherian ring. Show that the following are equivalent:
  (a) R is artinian,
  (b) $\operatorname{Spec}(R)$ is discrete (i.e., every subset is open) and finite,
  (c) $\operatorname{Spec}(R)$ is discrete.
  Give an example of a noetherian ring R such that $\operatorname{Spec}(R)$ is finite, but R is not artinian.

(4) Let R be a principal ideal domain and M a finitely generated R-module. Explain how to calculate the associated primes of M in terms of the structure theorem from HW2#4.

(5) Let k be a field and $R = k[x_1, \dots, x_n]$ a polynomial ring. A **monomial** is an element of the form $x_1^{d_1} \cdots x_n^{d_n}$ for some $d_i \geq 0$. A **monomial ideal** is an ideal generated by monomials.
  (a) Describe which monomial ideals are prime.
  (b) Describe which monomial ideals are radical.
  (c) Describe which monomial ideals are primary.

2. Suggested exercises (don't submit)

From Altman–Kleiman:

- Chapter 16: 18, 20, 28, 29, 30
- Chapter 17: 6, 8, 11, 22
- Chapter 18: 6, 7, 8

[1] It follows easily from what we've seen that every noetherian ring is coherent.
[2] For a definition, see Altman–Kleiman, Example 3.10.