Math 222 (Steven Sam), Fall 2016
Homework 9, due November 23

Here is a property about sequences that I didn't state in class, but you can use:

Theorem 1. *Let $f(x)$ be a continuous function and define a sequence by $a_k = f(k)$. If $\lim_{x\to\infty} f(x)$ exists, then $\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x)$.*

But be careful: the example we saw in class with $f(x) = \sin(\pi x)$ shows that $\lim_{n\to\infty} a_n$ might exist even if $\lim_{x\to\infty} f(x)$ does not.

Exercises

Only the starred problems (8 total) need to be submitted for grading.

Chapter 5.3 (page 102): 1, 2*, 5, 7, 8*, 9*

(E1)* Calculate $\lim_{n\to\infty} \frac{\ln n}{n}$. Now apply the exponential function to calculate $\lim_{n\to\infty} \sqrt[n]{n}$.

(E2)* Determine for which p the series $\sum_{k=2}^{\infty} \frac{1}{k(\log k)^p}$ converges.

(E3) For which values of x does $\sum_{k=0}^{\infty} (3x)^k$ converge? How about $\sum_{k=0}^{\infty} 3x^k$?

(E4)* Which of the following alternating series converge? Why?

(a) $\sum_{k=1}^{\infty} (-1)^k k$

(b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(c) $\sum_{k=1}^{\infty} (-1)^k \frac{k+1}{k}$

(E5)* Which of the following series converge? Why?

(a) $\sum_{k=1}^{\infty} \frac{5}{2+3^k}$

(b) $\sum_{k=1}^{\infty} \frac{k}{(k+1)2^k}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{1+2^k}{1+3^k}$

(E6)* Give an example of a convergent series $\displaystyle\sum_{k=1}^{\infty} a_k$ and a divergent series $\displaystyle\sum_{k=1}^{\infty} b_k$ such that $a_n \geq b_n$ for all n.

(E7) Define $f(x) = |\sin(\pi x)|$. Explain why $\displaystyle\sum_{k=1}^{\infty} f(k)$ converges and $\displaystyle\int_1^{\infty} f(x)dx$ diverges. Why doesn't this contradict the integral comparison test?