CHAPTER

7

Cyclic Phenomena: The Six Basic Trigonometric Functions

7.1 Angles

The size of an angle can be expressed in several ways. You've used "degrees" for years. They are simple. You've also used "revolutions" when talking about the revolutions per minute (RPM) of an engine. In calculus, "radians" are used as a unit of measure because the rules of calculus are easiest that way. How big is a radian? Here's how big: **it's the angle corresponding to an arc length of 1 in a unit circle.** Look at Figure 7.1. A "unit circle" indicates that the radius = 1, and we'll always put the center at (0, 0) for convenience. The angle θ as drawn is 1 radian, because the arc length "subtended" (cut off) by the angle has length = 1. By the way, θ is the Greek letter theta and is often used for angles.

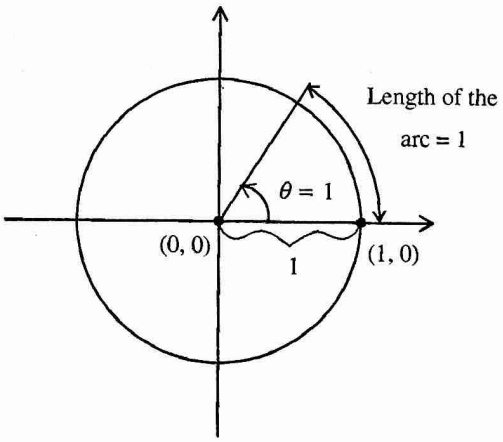


Figure 7.1

Notice that if you double the length of the arc, you double the angle. In fact, in a unit circle, the arc length equals the angle (if you use radians). Since most of you are probably more comfortable using degrees, we must deal with the key question: how do you convert from degrees to radians, and vice versa? Here's the secret:

One revolution = the complete angle at the center in radians, so
= the length of the complete arc
= the circumference of the unit circle
= $2\pi r$, but $r = 1$, so
= 2π (radians, that is)

You recall that there are 360 degrees in one full revolution, so

$$\boxed{360° = 2\pi \text{ radians}}.$$

If you divide both sides of this equation by 2π, you obtain

$$1 \text{ radian} = \frac{360°}{2\pi} \cong 57°,$$

while dividing both sides of the equation by 360 gives

$$1° = \frac{2\pi}{360} \text{ radians} = \frac{\pi}{180} \text{ radians} \cong 0.017 \text{ radians}.$$

We will **always** express angles in **radians**, unless specified differently. Moreover, angles are measured starting at the positive x-axis going counterclockwise if the angle is positive, clockwise if the angle is negative. Typically, we will also leave π simply as π, and will not use any decimal approximation. To a mathematician, leaving π in an answer is fine. In fact, it's better than fine because it's exact. Anything else would be less exact. (However, if you go to a hardware store and ask the clerk for π square inches of sheet metal, you will get (and deserve!) some funny stares.)

EXAMPLE 1 Convert 27° to radians.

Solution $360° = 2\pi$ (If things are clear, we usually drop the word *radians*.)

$$1° = \frac{2\pi}{360}, \text{ so } 27° = 27 \cdot \frac{2\pi}{360} = \frac{3}{40} \cdot 2\pi = \frac{3\pi}{20} \quad \blacksquare$$

EXAMPLE 2 Convert $-\frac{5\pi}{6}$ radians to degrees, and diagram it.

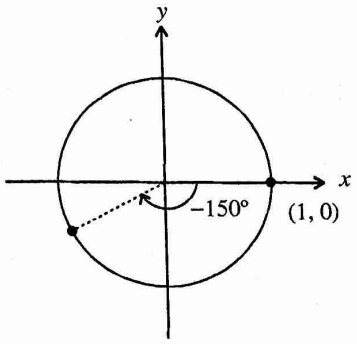
Solution

$$2\pi = 360°$$

$$\pi = 180°$$

$$-\frac{5}{6} \cdot \pi = -\frac{5}{6} \cdot 180° = -150°$$

The angle is measured clockwise, because it is negative, and is shown in the following figure.



$\blacksquare$

You should commit to memory the following conversion chart since these angles will come up again and again.

Degrees	360	180	90	60	45	30
Radians	2π	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

7.1 Exercises

1) Change to radian measure:

a) $120°$ b) $270°$

c) $135°$ d) $210°$

e) $-150°$ f) $450°$

2) Change the following angles from degree to radian measure:

a) $75°$ b) $225°$

c) $130°$ d) $300°$

3) Change to degrees:

a) $\frac{3\pi}{4}$ b) $\frac{11\pi}{6}$

c) $-\frac{\pi}{3}$ d) 3π

e) $\frac{9\pi}{2}$ f) $\frac{9\pi}{4}$

4) Change the following angles from radians to degrees:

a) $\frac{5\pi}{6}$ b) $\frac{5\pi}{3}$

c) $-\frac{2\pi}{3}$ d) 34π

5) Draw a large circle and mark each of the angles in Exercise 3.

6) Draw a large circle and mark each of the angles in Exercise 4.

7.2 Definition of $\sin\theta$ and $\cos\theta$

Consider the unit circle, centered at the origin, with an angle of θ radians, as shown in Figure 7.2.

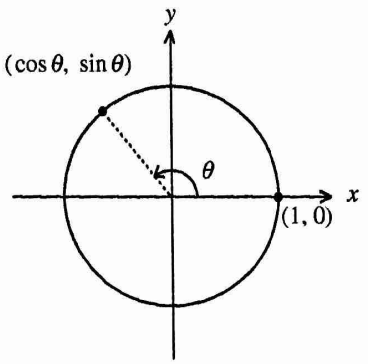


Figure 7.2

(Notice that the angle is measured from the positive x-axis, counterclockwise.) The dotted line defining the terminal side (end) of the angle θ intersects the circle at a point. As the angle θ changes, so do the coordinates of that point, so each of the coordinates is a function of the angle θ. These two functions are very important, and so they have their own names.

DEFINITION

In Figure 7.2 the first coordinate is called **cos θ** (short for cosine of θ). The second coordinate is called **sin θ** (short for sine of θ).

Remarks

a) Since this point is on the unit circle, its coordinates must satisfy the equation of that circle: $x^2 + y^2 = 1$, that is, $(\cos\theta)^2 + (\sin\theta)^2 = 1$.

b) To avoid the constant use of parentheses, we write $\cos^n\theta$ to mean $(\cos\theta)^n$; similarly, we write $\sin^n\theta$ to mean $(\sin\theta)^n$. Thus $\cos^2\theta + \sin^2\theta = 1$. We usually write this equation as $\sin^2\theta + \cos^2\theta = 1$.

c) Imagine θ going through values from 0 to 2π. Then the point on the circle goes from (1, 0) counterclockwise around the circle. Thus, its second coordinate, $\sin\theta$, which is the altitude of the point, goes first from 0 up to 1, then down to -1, then back up to 0, as shown in the graph in Figure 7.3.

Meanwhile, the first coordinate, $\cos\theta$, goes from 1 down to -1 and back to 1, as shown in Figure 7.4.

d) Since θ and $\theta + 2\pi$ correspond to the same point on the unit circle, we have $\sin\theta = \sin(\theta + 2\pi)$. This means that the graphs repeat over and

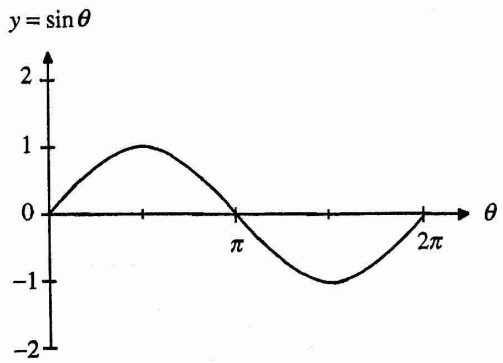


Figure 7.3

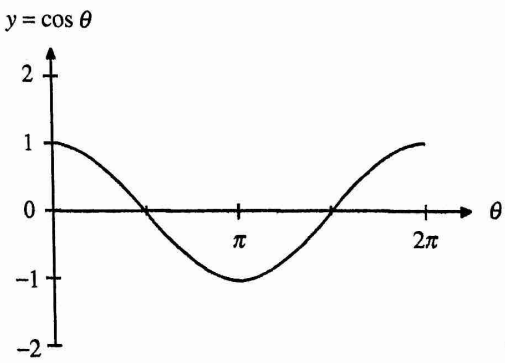


Figure 7.4

over ad infinitum. In fact, $\sin\theta = \sin(\theta + 2k\pi)$ for any whole number k. We say that $\sin\theta$ is **periodic** with period 2π. The same is true of $\cos\theta$. Remember that $2\pi k$ radians, where k is a whole number, is exactly k revolutions.

e) We just happened to use the variable name θ. We could also use x, y, z, w, u, v, ☺, ☹, or any other letter or symbol desired! To be consistent with the function notation $f(x)$ we've come to know and love, we will use the variable x. If we allow x to be any real number, we get the **full** graph of $f(x) = \sin x$ and $g(x) = \cos x$, shown in Figures 7.5 and 7.6.

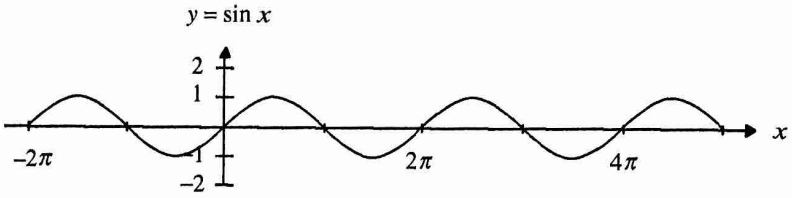


Figure 7.5

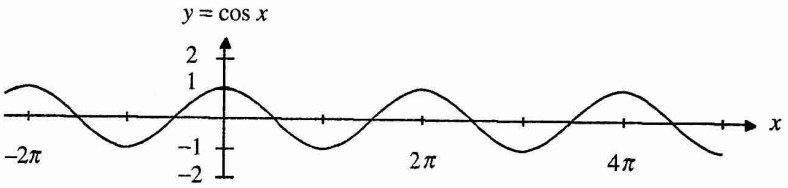
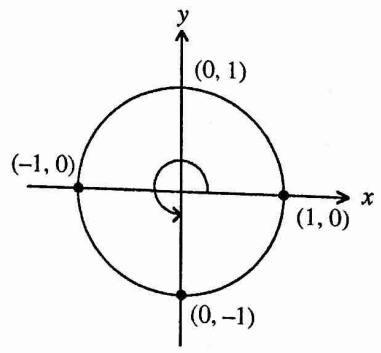


Figure 7.6

f) You might guess that $\sin x$ and $\cos x$ and other trigonometric functions would be useful for studying waves, daily temperature averages through the seasons, alternating current, electronic signals, and lots of other **cyclic phenomena**. You'd be perfectly right. These functions are the basic tools of the trade. Wave-shaped graphs are said to be **sinusoidal**.

EXAMPLE 1 Evaluate $\cos\frac{3\pi}{2}$ and $\sin\frac{3\pi}{2}$.

Solution To determine the sine and cosine of some angles is really quite easy. Just remember the following diagram. The angle $\frac{3\pi}{2}$ is 3/4 of the way around the circle, ending up at the point $(0, -1)$. For angles that end up on the coordinate axes, the sine and cosine are given immediately because we know exactly what the coordinates of those points are.



Beginning with the point (1, 0), and traveling counterclockwise, the four points labeled in the preceding figure are the endpoints for the angles 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π, whose point is the same as the point corresponding to angle 0.

In this case, for $\frac{3\pi}{2}$ the endpoint has an x-coordinate of 0, and a y-coordinate of -1. Just remember ***x*** before ***y***, **c** before **s**. The x-coordinate is the **c**osine, and the y-coordinate is **s**ine. So

$$\cos\frac{3\pi}{2} = 0 \text{ and } \sin\frac{3\pi}{2} = -1. \quad \blacksquare$$

EXAMPLE 2 What is $\cos\frac{9\pi}{2}$?

Solution First of all, $\frac{9\pi}{2} = 4\pi + \frac{\pi}{2}$, and 4π radians equals two complete revolutions. So the angle $4\pi + \frac{\pi}{2}$ goes twice around the circle (which does nothing), and then another 90°, ending up at the top of the circle, at the point (0, 1). The cosine is the first coordinate of that point. So

$$\cos\frac{9\pi}{2} = 0. \quad \blacksquare$$

Alternative definition of sin θ and cos θ: The definition we just saw for sin θ and cos θ is called the **unit circle** definition. It is valid for all real numbers θ. However, if $0 < \theta < \frac{\pi}{2}$, there is another, equivalent, definition, called the **right-angled triangle** definition, with which you are probably more familiar. To illustrate, consider any number θ with $0 < \theta < \frac{\pi}{2}$. Notice that θ radians would be an acute angle, so you may draw a right triangle (short for right-angled triangle) with an angle of θ radians, as, for example, ΔABC. See Figure 7.7.

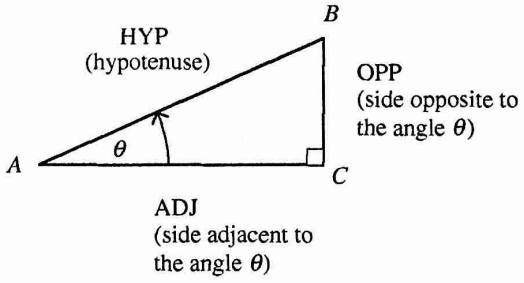


Figure 7.7

Now place the unit circle onto ΔABC, with its center at A and the positive x-axis along the line segment AC. See Figure 7.8.

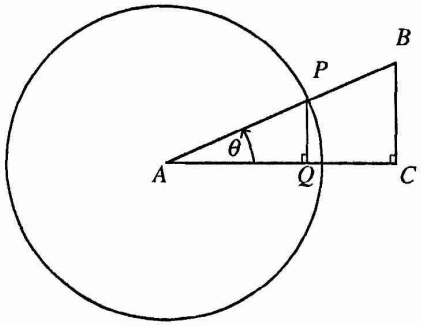


Figure 7.8

Let the intersection of the circle and the line AB be called P. From P drop the perpendicular to AC, to a point Q. Since P is on the unit circle and the terminal line of the angle θ, we have $P = (\cos\theta, \sin\theta)$. Hence, $PQ = \sin\theta$, while $AQ = \cos\theta$ and $AP = 1$. So here we have the two triangles in Figures 7.9(a) and 7.9(b), which are in fact similar.

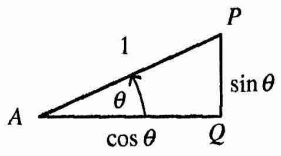


Figure 7.9(a)

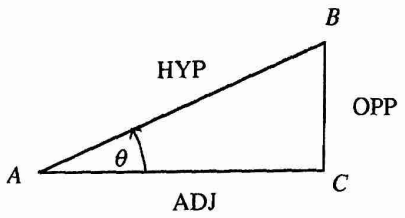


Figure 7.9(b)

Since they are similar, their sides are in proportion, so

$$\frac{\sin\theta}{1} = \frac{\text{OPP}}{\text{HYP}},$$

and hence

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}}.$$

Similarly

$$\frac{\cos\theta}{1} = \frac{\text{ADJ}}{\text{HYP}},$$

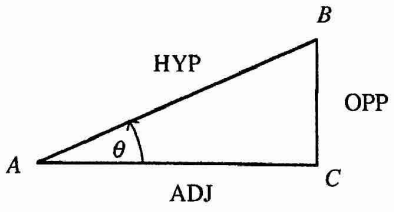
and so

$$\cos\theta = \frac{\text{ADJ}}{\text{HYP}}.$$

DEFINITION

So we have the following alternative definition for $0 < \theta < \frac{\pi}{2}$.

Let ΔABC be any right-angled triangle with angle A equal to θ.



Then $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{\text{BC}}{\text{AB}}$,

and $\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}} = \dfrac{\text{AC}}{\text{AB}}$.

7.2 Exercises

1) Evaluate:

a) $\sin\dfrac{7\pi}{2}$

b) $\cos\dfrac{5\pi}{2}$

c) $\cos\dfrac{-9\pi}{2}$

d) $\sin 101\pi$

2) Evaluate:

a) $\cos 12\pi$

b) $\sin\dfrac{5\pi}{2}$

c) $\sin\dfrac{-9\pi}{2}$

d) $\cos 101\pi$

3) Assuming k is a whole number, evaluate the following:

   a) $\sin(\pi/2 + 2k\pi)$

   b) $\cos(-\pi/2 + 2k\pi)$

   c) $\sin k\pi$

   d) $\cos k\pi$

4) What is $\cos(\theta + \pi)$ in terms of $\cos\theta$? (*Hint*: Use the unit circle.)

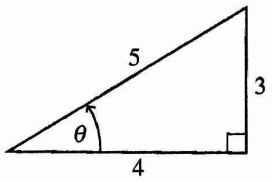
5) Assuming k is a whole number, evaluate the following:

   a) $\cos(\pi/2 + 2k\pi)$

   b) $\sin(-\pi/2 + 2k\pi)$

6) a) In the triangle shown, calculate $\sin\theta$ and $\cos\theta$.

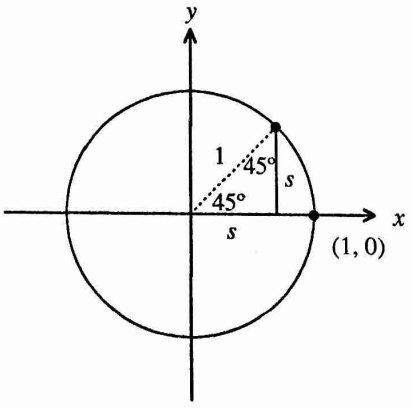
   b) Calculate $\sin^2\theta + \cos^2\theta$.



7.3 Special Angles $(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{3})$

EXAMPLE 1 Find $\sin\frac{\pi}{4}$.

Solution Since $\frac{\pi}{4} = 45°$, the graph looks like



where s is some number.

Hence, the coordinate of the point in question is (s, s), and $s = \sin\frac{\pi}{4} = \cos\frac{\pi}{4}$. Since $\sin^2 x + \cos^2 x = 1$, we have

$$\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = s^2 + s^2 = 2s^2 = 1.$$

Hence

$$s^2 = \frac{1}{2} \text{ and } s = \pm\frac{1}{\sqrt{2}}.$$

Since it is clear that $s > 0$, we have $s = \frac{1}{\sqrt{2}}$, and so:

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}. \quad \blacksquare$$

Remark **Memorize the triangle in Figure 7.10.** (It will simplify your life.)

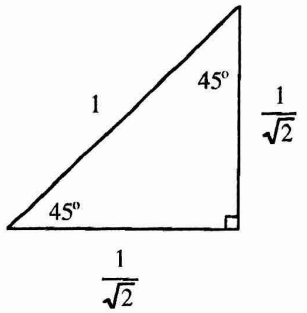
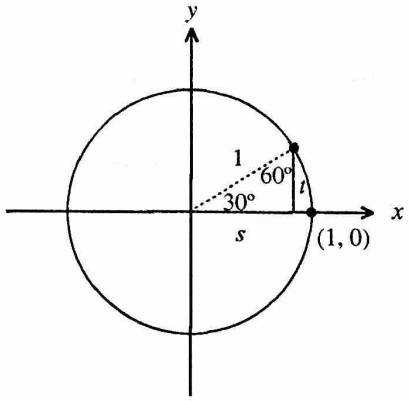


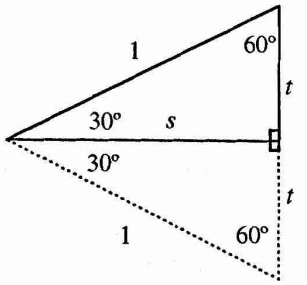
Figure 7.10

EXAMPLE 2 Find $\sin\frac{\pi}{6}$ and $\cos\frac{\pi}{6}$.

Solution Since $\frac{\pi}{6} = 30°$, the graph now looks like this:



How big is s? And t? Here's the trick: flip down the triangle along its base, as shown.



Notice that the result is an equilateral triangle, so the vertical side is also 1. This means $2t = 1$, or $t = \frac{1}{2}$. Now since

$$s^2 + t^2 = 1,$$

$$s^2 + \frac{1}{4} = 1,$$

$$s^2 = \frac{3}{4},$$

and

$$s = \pm\frac{\sqrt{3}}{2}.$$

Again, it is clear that $s > 0$, which implies

$$\sin\frac{\pi}{6} = \frac{1}{2} \text{ and } \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}. \quad \blacksquare$$

Remark **Memorize the triangle in Figure 7.11 also.** (This is the last one—we promise!)

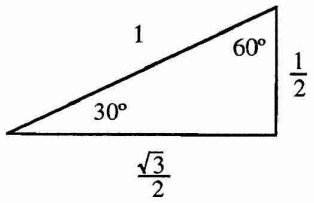
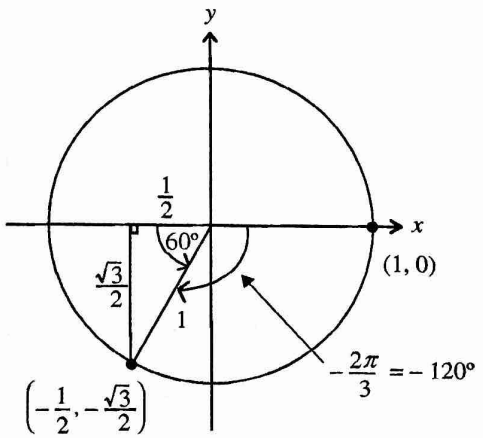


Figure 7.11

The two triangles we asked you to memorize can be used to determine trigonometric values for many more angles. Consider the following example.

EXAMPLE 3 Find $\sin\dfrac{-2\pi}{3}$.

Solution Since $-\frac{2\pi}{3} = -120°$, the triangle in the following figure is exactly the 30°, 60°, 90° triangle shown in Figure 7.11.



Hence, $\sin\frac{-2\pi}{3} = -\frac{\sqrt{3}}{2}$. (See the diagram.) $\blacksquare$

7.3 Exercises

Evaluate the following:

1) $\sin\frac{3\pi}{4}$

2) $\sin\frac{7\pi}{4}$

3) $\cos\frac{-\pi}{6}$

4) $\cos\frac{-3\pi}{4}$

5) $\cos\frac{-11\pi}{4}$

6) $\sin\frac{-5\pi}{4}$

7) $\sin\frac{2\pi}{3}$

8) $\cos\frac{13\pi}{6}$

9) $\sin\frac{4\pi}{3}$

10) $\cos\frac{-5\pi}{3}$

11) $\cos\frac{7\pi}{6}$

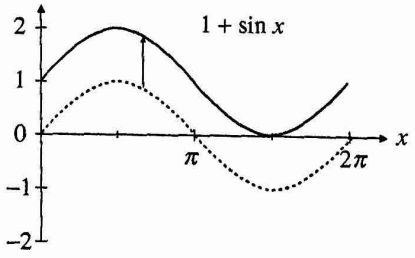
12) $\sin\frac{29\pi}{6}$

7.4 Graphs Involving sin *x* and cos *x*

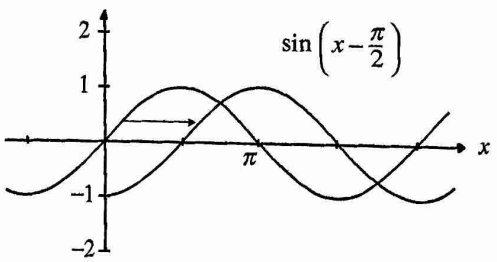
Remember Chapter 4, where we shifted graphs up and down, left and right, and so on? We can, of course, do that with sin x and cos x also.

EXAMPLE 1 On $[0, 2\pi]$, graph $\sin x$, $1 + \sin x$, and $\sin\left(x - \frac{\pi}{2}\right)$.

Solution The graph of $1 + \sin x$ is that of $\sin x$ shifted up a distance of 1. Recalling Figure 7.3, we obtain the graph:



The graph of $\sin\left(x - \frac{\pi}{2}\right)$ is that of $\sin x$ shifted right a distance of $\frac{\pi}{2}$.



■

A shift to the left or right of a wave-shaped graph, such as sin x or cos x, is called a **phase shift**.

Now for vertical scaling, or stretching and compressing, consider the graph of $y = \sin x$ on the interval $[0, 2\pi]$ shown in Figure 7.12. How do you stretch it away from the x-axis so it is twice as high above the axis and twice as low below the axis? Simple: y needs to be twice as big, so $y = 2 \sin x$ will do it.

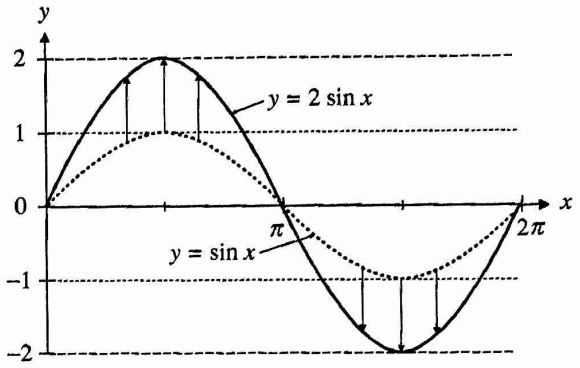


Figure 7.12

To compress the graph so that it lies between $y = -\frac{1}{2}$ and $y = \frac{1}{2}$, simply multiply the function sin x by $\frac{1}{2}$. Figure 7.13 illustrates $y = \frac{1}{2}\sin x$.

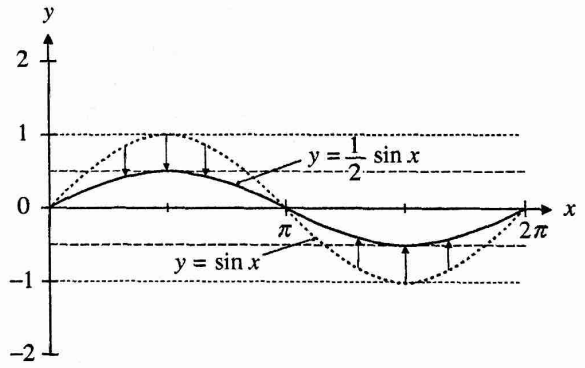


Figure 7.13

In general, when given the graph of $f(x)$, the graph of $k \cdot f(x)$ is that of $f(x)$ stretched if $k > 1$ and compressed if $0 < k < 1$. What happens if $k < 0$? Well, let's see. The graph of $y = -\sin x$ is shown in Figure 7.14.

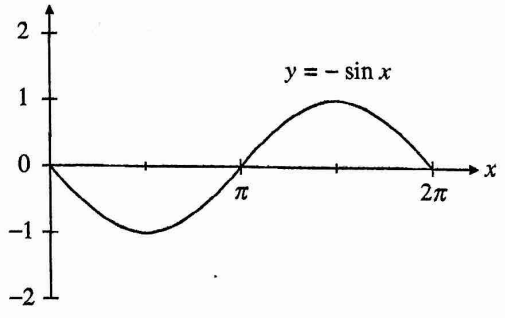


Figure 7.14

It's the mirror reflection of the graph of $y = \sin x$ about the x-axis. In general, for $k < 0$, the graph of $k \cdot f(x)$ is obtained from the graph of $f(x)$ through reflection

about the x-axis and a scaling depending on the size of $|k|$. For example, the graph of $-2f(x)$ is obtained by taking the graph of $f(x)$, stretching by a factor of 2, and then reflecting about the x-axis. The graph of $-\frac{1}{2}f(x)$ is obtained by taking the graph of $f(x)$, shrinking it to $\frac{1}{2}$ its vertical size, and then reflecting about the x-axis.

The **amplitude** of a sinusoidal graph is equal to $\frac{1}{2}$ of the distance from the top to the bottom of the waves. For example, $y = 2 \sin x$ has amplitude 2, and $y = 1 - 3 \sin x$ has an amplitude of 3.

Now let's consider horizontal stretching and compressing. When graphing sin x, as x goes from 0 to 2π, sin x goes from 0 to 1, to -1, and back up to 0, as shown in Figure 7.15.

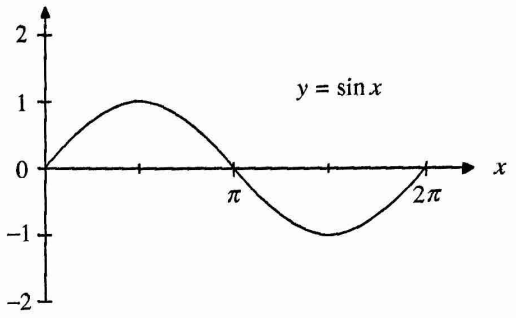


Figure 7.15

(sin x is said to go through **one cycle** on $[0, 2\pi]$, because the rest is repetition.) Old news! Ho hum! Now, what happens to sin $2x$? Well, as x goes from 0 to 2π, $2x$ goes from 0 to 4π, and the sine of it (i.e., sin $2x$) will have the chance to go through two cycles, as in Figure 7.16.

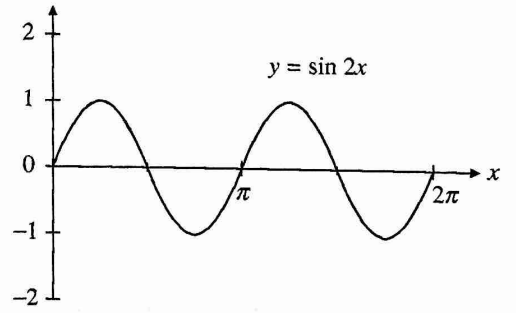


Figure 7.16

On the other hand, as Figure 7.17 shows, for $\sin\frac{1}{2}x$ as x goes from 0 to 2π, $\frac{1}{2}x$ goes from 0 to π (i.e., through only half a cycle).

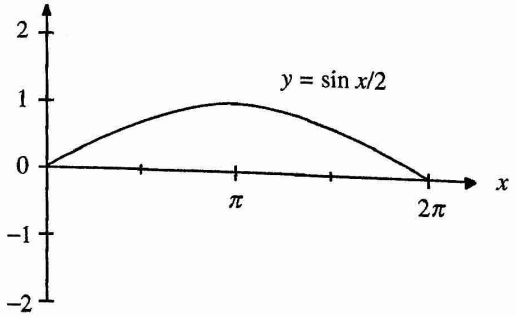


Figure 7.17

In general, here's how you go from $y = \sin x$ to $y = \sin ax$. (In fact, the same method holds in general, for going from $y = f(x)$ to $y = f(ax)$.)

1) If $a > 1$, you compress the graph toward the y-axis. If a is a positive integer, then the graph of $y = \sin ax$ has a complete **oscillations**, or cycles, in the interval $[0, 2\pi]$. (This is also true for $y = \cos ax$.)

2) If $0 < a < 1$, you stretch the graph away from the y-axis. The graph of $y = \sin ax$ has a complete oscillations, or cycles, in the interval $[0, 2\pi]$. For example, if $a = \frac{1}{5}$, we would have one-fifth of the total cycle.

3) If $a < 0$, the graph is reflected about the y-axis, and then compressed or stretched by a factor of $|a|$.

Don't forget:

1) You can't "factor out" the 2 and say that $\sin(2x)$ and $2\sin(x)$ are the same thing even if the devil tempts you to. (How simple life would be!) Test it with $x = \frac{\pi}{2}$.

2) You also can't say that $\sin(x + y)$ and $\sin x + \sin y$ are equal, except by lying or fooling yourself.

Remark We saw that sinusoidal graphs are made of cycles that just keep repeating. The length of such a cycle is called the **period** of the function, and the reciprocal of the period is called the **frequency**. So $\sin x$ has a period of 2π and a frequency of $\frac{1}{2\pi}$, while $\sin 3x$ has a period of $\frac{2\pi}{3}$ and a frequency of $\frac{3}{2\pi}$.

7.4 Exercises

1) From the graph of $y = \sin x$, graph the following on $(0, 2\pi)$:

a) $y = 3\sin x$ b) $y = -\frac{1}{2}\sin x$

c) $y = -2\sin x$

2) From the graph of $y = \sin x$, graph the following on $(0, 2\pi)$:

a) $y = 2.03\sin x$ b) $y = -\frac{1}{4}\sin x$

c) $y = -4\sin x$

3) Graph the following functions over the interval $(0, 2\pi)$:

a) $y = \sin 4x$ b) $y = \cos 3x$

c) $y = \sin\frac{1}{4}x$

4) Graph the following functions over the interval $(0, 2\pi)$:

a) $y = \sin 2x$ b) $y = \cos\frac{3}{2}x$

c) $y = \sin \pi x$

5) Graph all of the following functions on $(0, 2\pi)$ on the same set of axes.

a) $y = 2\sin x$ b) $y = 2\sin(x - \pi)$

c) $y = 2\sin(x - \pi) + 5$

6) Graph all of the following functions on $(0, 2\pi)$, on the same set of axes.

a) $y = 2\cos x$

b) $y = 2\cos 2x$

c) $y = 2\cos 2x + 2$

7) On $(0, 4\pi)$ graph $y = 1 - 2\cos x$.

8) On $(-2\pi, 4\pi)$ graph $y = 1 - 3\sin x$.

9) Graph $y = 1 + 2\cos 3x$, and find its amplitude, period, and frequency.

10) Graph $y = 3\sin(2x) - 1$, and find its amplitude, period, and frequency.

11) Graph $y = \sin 2\pi x$, and find its period and frequency.

12) Graph $y = 5\sin(3x) + 2$, and find its period and frequency.

13) Graph $\sin x$ on $(0, 2\pi)$ and $\sin(-x)$ on $(-2\pi, 0)$.

14) The graph of $f(x) = A\sin x$ oscillates between A and $-A$, taking on those particular values when $x = \frac{\pi}{2} + 2k\pi$ and $x = \frac{3\pi}{2} + 2k\pi$, respectively. What if A is not constant but a function of x? What do you think the graph of $g(x) = A(x)\sin x$ does? Graph $\sqrt{x}$ and $\sqrt{x}\sin x$.

7.5 The Other Trigonometric Functions

Having defined the functions $f(x) = \sin x$ and $f(x) = \cos x$ for any real number x, there are four other basic trigonometric functions that you will encounter on your travels:

1) $\tan x = \dfrac{\sin x}{\cos x}$ (called **tangent**)

2) $\cot x = \dfrac{\cos x}{\sin x}$ (called **cotangent**)

3) $\sec x = \dfrac{1}{\cos x}$ (called **secant**)

4) $\csc x = \dfrac{1}{\sin x}$ (called **cosecant**).

Remark At certain values of x, the denominator is zero; hence, at those values of x, that function is not defined. For example, $\tan x$ is not defined at $\frac{\pi}{2}$, because $\cos\frac{\pi}{2} = 0$.

7.5 Exercises

1) Evaluate, if they are defined:

a) $\tan\dfrac{5\pi}{2}$

b) $\csc\dfrac{5\pi}{2}$

c) $\sec\dfrac{5\pi}{2}$

d) $\cot\dfrac{5\pi}{2}$

e) $\tan\frac{-\pi}{2}$

f) $\sin\frac{2\pi}{3}$

g) $\sec\frac{-\pi}{2}$

h) $\cot\frac{-\pi}{2}$

2) Evaluate:

a) $\csc\frac{3\pi}{4}$

b) $\tan\frac{5\pi}{6}$

c) $\cot\frac{-2\pi}{3}$

d) $\sec\frac{7\pi}{4}$

e) $\cot\frac{-\pi}{3}$

f) $\sec\frac{-3\pi}{4}$

g) $\tan\frac{5\pi}{4}$

h) $\csc\frac{2\pi}{3}$

3) Evaluate:

a) $\tan\frac{3\pi}{4}$

b) $\csc\frac{5\pi}{6}$

c) $\sec\frac{-2\pi}{3}$

d) $\cot\frac{7\pi}{4}$

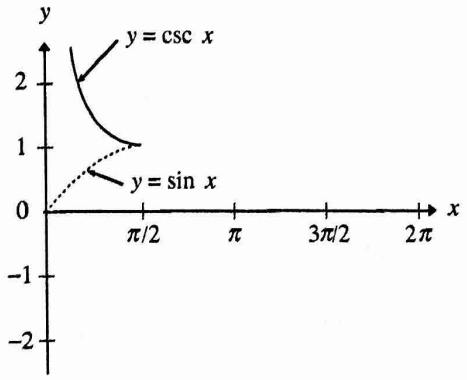
e) $\tan\frac{-\pi}{3}$

f) $\csc\frac{-3\pi}{4}$

g) $\sec\frac{5\pi}{4}$

h) $\cot\frac{2\pi}{3}$

4) Notice that $\frac{1}{x}$ gets large if x gets small—for example, $\frac{1}{\frac{1}{1000}} = 1000$. Also, recall that $\csc x = \frac{1}{\sin x}$. Thus, if we draw the graph of $\sin x$ from 0 to $\frac{\pi}{2}$, we can envision the graph of $\csc x$ as shown in the following figure. (When $\sin x$ is small, $\csc x$ is large. As $\sin x$ gets close to 0, $\csc x$ increases without bound.)



Complete this sketch for x in $[0, 2\pi]$—i.e., graph $\csc x$ on $[0, 2\pi]$ (except at certain points where it is not defined).

5) Similarly, graph $\sec x$ on $[0, 2\pi]$ and then for several periods (except at certain ...).

6) Graph $\tan x$ on $[-\pi, \pi]$ (except at certain ...).

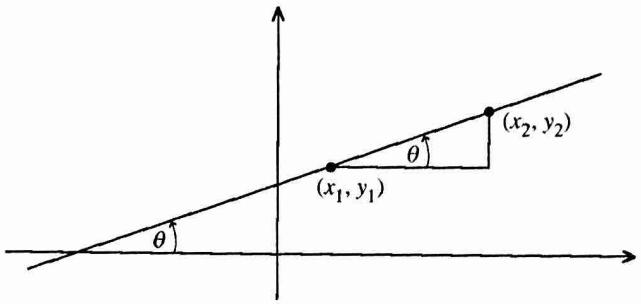
7) Graph $\cot x$ on $[0, 2\pi]$ (except at certain ...).

8) Using the unit circle, estimate $\cos(-1.2)$. (A rough approximation is fine.)

(*Hint:* $\frac{\pi}{2} \approx \frac{3}{2} = 1.5$, so 1.2 is an angle a little smaller than 90°.)

9) Estimate sin(3.0).

10)



Consider the line L with equation $y = mx + b$ and angle of inclination θ as shown.

a) Calculate $\tan\theta$ in terms of the coordinates shown.

b) Calculate the slope m of the line in terms of those coordinates.

c) Notice anything?



7.6 Graphs of Complex Trigonometric Functions

Section 7.4 showed how the graphs of sines and cosines (also called "sine waves") can be moved up and down, left and right, and stretched and compressed, horizontally and vertically. Here is an additional level of complexity, combining some of these moves. You may verify the results on your graphing calculator.

EXAMPLE 1 On $[0, 2\pi]$, sketch the graph of $\sin(2x - \pi)$.

Solution Notice that $\sin(2x - \pi) = \sin\left(2\left[x - \frac{\pi}{2}\right]\right)$. Using the ideas of Section 7.4, we see that:

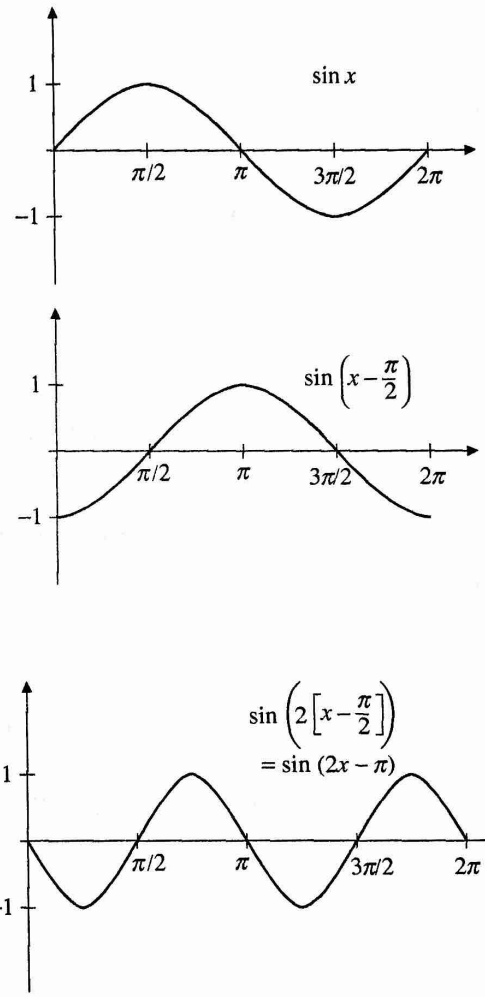
1) To go from $\sin x$ to $\sin\left(x - \frac{\pi}{2}\right)$, we shift the graph of $\sin x$ to the right a distance of $\frac{\pi}{2}$.

2) To go from $\sin y$ to $\sin 2y$, we compress the graph of $\sin y$ horizontally toward $y = 0$ by a factor of 2. Hence, to go from the graph of $\sin\left(x - \frac{\pi}{2}\right)$ to the graph

of $\sin\left(2\left[x - \frac{\pi}{2}\right]\right)$, we compress the graph of $\sin\left(x - \frac{\pi}{2}\right)$ horizontally by a factor of 2 toward $x - \frac{\pi}{2} = 0$, that is, $x = \frac{\pi}{2}$.

3) To sum up: we take the graph of $\sin x$, shift it to the right a distance of $\frac{\pi}{2}$, then horizontally compress the graph by a factor of 2 toward $x = \frac{\pi}{2}$.

Here is the sequence:



7.6 Exercises

1) On $[0, 4\pi]$, graph $\sin\left(\frac{x + \pi}{2}\right)$ by first graphing $\sin x$ and $\sin(x + \pi)$.

2) On $[0, \pi]$, graph $\cos(4x - \pi)$ and the two earlier graphs of this method.

3) On $[0, \pi]$, graph $\tan\left(\frac{1}{2}x - \pi\right)$ and the two earlier graphs of this method.