MATH2 Additional handout

Factoring quadratic expressions

A quadratic expression is an expression of the form ax^2+bx+c when $a \neq 0$. The goal of this handout is to provide an algorithm to factor ax^2+bx+c to $a(x-p)(x-q)$.

Step 1 Factor out a. That is, write the expression as $a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$. If either$\frac{b}{a}$ or $\frac{c}{a}$ are not integers, do not proceed. Just use formula.

Step 2 Then, we will have a quadratic expression of the form x^2+bx+c. Here, we have two cases:

Step 2.1.1: If c is positive: Ignore the sign of b and c. We want to find two numbers, call them p, q, such that

- Their sum is b.
- Their product is c.

Step 2.1.2 Now, x^2+bx+c will look like $(x__p)(x__q)$. To fill the signs, both places will have the same sign, which will be the same sign as b.

Step 2.2.1: If c is negative: Ignore the sign of b and c. We want to find two numbers, call them p, q, such that

- Their difference is b.
- Their product is c.

Step 2.2.2 Now, x^2+bx+c will look like $(x__p)(x__q)$. To fill the signs, two places will have different sign. Assign the same sign as b to the bigger number between p and q.

Lastly, do not forget a, if you factor it out at the beginning.

Example 1 Factorize $2x^2 + 8x + 6$:

First we factor out 2, so we have $2(x^2+4x+3)$. So, we instead look at x^2+4x+3. This falls into the first case.

Two numbers whose product is 3 and sum is 4 are 3 and 1. So, we have $(x__3)(x__1)$. They should have the same sign, which is the same sign as 4, so plus sign. So, $x^2 + 4x + 3 = (x + 3)(x + 1)$. Thus,

$$2x^2 + 8x + 6 = 2(x + 3)(x + 4)$$

Example 2 Factorize $3x^2 - 6x - 24$:

$3x^2 - 6x - 24 = 3(x^2 - 2x - 8)$. To factor $x^2 - 2x + 8$, we are in the second case.

Two numbers whose produce is 8 and difference is 2 are 4 and 2. So, we have $(x__4)(x__2)$. 4 is the bigger number, so it takes the sign of -2. Thus, we have $(x - 4)(x + 2)$. Thus,

$$3x^2 - 6x - 24 = 3(x - 4)(x + 2)$$