1. Fractions. Let a, b, c, and d be numbers.

   (a) You can break up a fraction from a sum in the numerator, but *not* in the denominator:

   $$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

   but

   $$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

   (b) Cancellation of the c here requires that it appears in *each* additive term of the numerator and denominator:

   $$\frac{ca+cb}{cd} = \frac{c(a+b)}{cd} = \frac{a+b}{d}$$

   but

   $$\frac{ca+b}{cd} \neq \frac{a+b}{d}$$

   (c) Compound fractions can be simplified by using the rule "division is the same as multiplication by the reciprocal":

   $$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

2. Natural Logs. Let a and b be numbers.

   (a) Natural logs distribute in a funny way over products and quotients:

   $$\ln(ab) = \ln a + \ln b$$

   $$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

   but they do *not* distribute over sums:

   $$\ln a + b \neq \ln a + \ln b$$

   (b) Natural logs can help you work with exponents by "bringing them down":

   $$\ln(a^b) = b \ln a$$

3. Exponents. Let a, b, m, and n be numbers.

   (a) Exponents distribute over products, but *not* over sums:

   $$(ab)^n = a^n b^n$$

   but

   $$(a+b)^n \neq a^n + b^n$$

   (b) A negative exponent can always be viewed as a denominator, and vice versa:

   $$a^{-n} = \frac{1}{a^n}$$

   (c) Two terms with exponents can only be multiplied if they share the same base; in that case, the exponents add:

   $$a^m a^n = a^{m+n}$$

   but $a^m d^n$ cannot be further simplified, and

   $$a^m a^n \neq a^{mn}$$

   (d) Similarly for division:

   $$\frac{a^m}{a^n} = a^{m-n}$$

4. Roots. Let a, b, m, and n be numbers.

   (a) Remember that roots can always be viewed as fractional exponents:

   $$\sqrt[n]{a} = a^{\frac{1}{n}}$$

   With this point of view, we'll inherit all the rules about exponents. In particular,

   (b) Distributing a root over a product:

   $$\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}} = \sqrt[n]{a}\sqrt[n]{b}$$

   (c) Multiplying two roots with a common base:

   $$\sqrt[m]{a}\sqrt[n]{a} = a^{\frac{1}{m}} a^{\frac{1}{n}} = a^{\frac{1}{m}+\frac{1}{n}}$$