SCAPDE FALL 2019 TITLES AND ABSTRACTS

Semyon Dyatlov

Title: Control of eigenfunctions on negatively curved surfaces.

Abstract: Given an L^2-normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set $\Omega \subset M$, what lower bound can we prove on the L^2-mass of the eigenfunction on Ω? The unique continuation principle gives a bound for any Ω which is exponentially small as $\lambda \to \infty$. On the other hand, microlocal analysis gives a λ-independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ-independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrödinger equation and exponential decay of damped waves. Joint work with Jean Bourgain, Long Jin, and Stéphane Nonnenmacher.

Mihaela Ifrim

Title:
Abstract:

Johnathan Luk

Title: High frequency limit in general relativity.

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Abstract: It is known in the physics literature that "high-frequency weak limits" of solutions to the Einstein vacuum equations are not necessarily vacuum solutions, but may have a non-trivial stress-energy-momentum tensor, which can be viewed physically as "effective matter fields" arising from back-reaction of high-frequency gravitational waves. Burnett conjectured nonetheless that any such limit is isometric to a solution to the Einstein-massless Vlasov system; and conversely that any solution to the Einstein-massless Vlasov system arises as such limit. We discuss some recent progress on Burnett's conjecture, as well as an application of it. This talk is based on joint works with Huneau, and with Rodnianski.

Dana Mendelson

Title: The focusing energy-critical nonlinear wave equation with random initial data.

Abstract: We consider the focusing energy-critical quintic nonlinear wave equation in three dimensional Euclidean space. It is known that this equation admits a one-parameter family of radial stationary solutions, called solitons. By randomizing radial initial data in $\dot{H}^s(\mathbb{R}^3) \times H^{s-1}(\mathbb{R}^3)$ for $s > 5/6$, which also satisfy a certain weighted Sobolev condition, we produce with high probability a family of radial perturbations of the soliton which give rise to global forward-in-time solutions of the focusing nonlinear wave equation that scatter after subtracting a dynamically modulated soliton. Our proof relies on a new randomization procedure using distorted Fourier projections associated to the linearized operator around a fixed soliton. To our knowledge, this is the first long-time random data existence result for a focusing wave or dispersive equation on Euclidean space outside the small data regime. Joint work with C. Kenig.

Kenji Nakanishi

Title: Failure of scattering with localized waves for the nonlinear Schrodinger equations with long-range interaction.

Abstract: This talk is based on joint work with Jason Murphy. We consider the nonlinear Schrodinger equation with the power or Hartree nonlinearity of long-range type and a time-dependent potential, and investigate if there is a global solution in the form of "Soliton Resolution" with the radiation part approximated by a free solution. We prove that such asymptotic behavior is impossible with non-trivial scattering in L^2, under very mild conditions on the non-radiative part and the potential. Those conditions allow not only multi-solitons and breathers, but also more general waves spreading slower than free ones.

Sung-Jin Oh

Title: On the Cauchy problem for the Hall-magnetohydrodynamics equations.
Abstract: In this talk, I will describe a recent series of work with I.-J. Jeong on the Cauchy problem for the Hall-MHD equation without resistivity. This PDE, first investigated by the applied mathematician M. J. Lighthill, is a one-fluid description of magnetized plasmas with a quadratic second-order correction term (Hall current term), which takes into account the motion of electrons relative to positive ions. Curiously, we demonstrate ill(!)posedness of the Cauchy problem near the trivial solution, despite the apparent linear stability and conservation of energy. On the other hand, we identify several regimes in which the Cauchy problem is well-posed, which includes the original setting that M. J. Lighthill investigated (namely, for initial data close to a uniform magnetic field). Central to our proofs is the viewpoint that the Hall current term imparts the magnetic field equation with a quasilinear dispersive character.

Benoit Pausader

Title: Asymptotic stability of the Minkowski space for the Einstein-Klein-Gordon equation.

Abstract: We present recent work with A. Ionescu on the stability of the simplest equilibrium in the Einstein-massive-scalar field system. This is one of the simplest model from general relativity that introduces a “matter-field” (where information propagates slower than the speed of light). One of the novel aspects is that we start with initial data which are rather general: the decay of the metric is slower than the

“mass term” 1/r and consistent with the assumptions on the second fundamental form (i.e. time slices are not necessarily maximal or “almost radially symmetric”).

Jared Speck

Title: A Remarkable Formulation of $3D$ Compressible Euler Flow and Applications.

Abstract: We recently derived a new formulation of the $3D$ compressible Euler equations with vorticity and entropy that sharply splits the flow into a geometric wave-part coupled to a div-curl-transport part. Both parts of the new formulation exhibit remarkable geometric and analytic structures, on par with the discovery of the good null structures in Einstein’s equations that led to breakthrough results in general relativity. For this reason, the new formulation has formed the starting point for a program on advancing the theory of compressible Euler flow without symmetry and without any irrotationality assumption. For example, it has formed the backbone for proving stable shock formation with vorticity. Most recently, it has played a crucial role in proving the existence of solutions having optimal regularity with respect to the “part of the flow” that blows up when shocks form, specifically, the wave-part. This can be viewed as the existence of “rough sound waves” coupled to vorticity and entropy transport. All of these results are intimately related and are also connected to other issues of interest such as Strichartz estimates, the regularity of sound cones, and the possibility of weakly continuing the solution past singularities. In this talk, I will survey these results and explain how they fit together, with an emphasis on rough sound waves. Various aspects of this program are joint with J. Luk, M. Disconzi, C. Luo, and G. Mazzone.

Daniel Tataru

Title: Low regularity water waves.

Abstract: The aim of this talk is to provide an overview of the low regularity well-posedness problem n the context of two dimensional gravity waves. These are quasilinear dispersive equations with a complex but very interesting structure. We show that this structure can be exploited in more than one way to drastically lower low regularity well-posedness thresholds. This is joint work with Albert Ai and Mihaela Ifrim.