

Name: POSSIBLE ANSWERS

Math 160A - Winter 2002 - Quiz #5

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Give examples (counter-examples) of structures which prove the following three statements. Be sure to describe explicitly the universe and the interpretation of each non-logical symbol.

1. $\exists x \forall y P(y, x) \not\equiv \exists x \forall y P(x, y)$.
2. $\forall x \exists y (y \leq x) \not\equiv \exists x \forall y (x \leq y)$.
3. $\forall x (f(x) \leq x) \not\equiv \exists x \forall y (x \leq y)$.

Possible answers:

1. Let $|\mathfrak{A}| = \{0, 1\}$ and $P^{\mathfrak{A}} = \{\langle 0, 1 \rangle, \langle 1, 1 \rangle\}$.
2. Let $\mathfrak{A} = (\mathbb{Z}, \leq)$ where \leq is the usual "less than or equal to".
3. Let $\mathfrak{A} = (\mathbb{Z}, \leq, f^{\mathfrak{A}})$ where \leq is the usual "less than or equal to", and $f^{\mathfrak{A}}(i) = i - 1$ for all i .