Name: ______________________________

Math 160A - Winter 2002 - Quiz #3 Answers

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Express each of the English sentences as a first-order formula. If the sentence is ambiguous, use your best judgement on what it means, and explain what is ambiguous and how you resolved the ambiguity. You may use our conventions on abbreviating formulas. The universe of objects is a population of people. Some of the people are politicians, and some people are honest. In the last election, each person voted for exactly one person.

The first order language has the following symbols: P and H are unary predicate symbols, and v is a unary function symbol. Equality ($=$) is also in the language. Their meanings are:

$P(x)$ - "x is a politician"

$H(x)$ - "x is honest"

$v(x)$ - "the person x voted for in the election"

1. Some politician is honest.

$$\exists x(P(x) \wedge H(x))$$

2. No politician is honest.

$$\forall x(P(x) \rightarrow \neg H(x))$$

3. Everyone is a politician.

$$\forall x(P(x))$$

4. Every politician voted from himself.

$$\forall x(P(x) \rightarrow v(x) = x)$$

5. Everyone who voted for himself is a politician.

$$\forall x(v(x) = x \rightarrow P(x))$$

6. No honest person voted for a politician.

$$\forall x(H(x) \rightarrow \neg P(v(x)))$$

7. One politician received all the votes.

$$\exists x(P(x) \wedge \forall y(v(y) = x))$$

8. No non-politician received any votes.

$$\forall x(\neg P(v(x)))$$

9. There were two politicians who voted for each other.

$$\exists x \exists y(x \neq y \wedge P(x) \wedge P(y) \wedge v(x) = y \wedge v(y) = x)$$

10. Everyone received at least one vote.

$$\forall x \exists y(v(y) = x)$$