Math 160A - Winter 2002 - Quiz #1′

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1. Prove or disprove each of the following. (A, B, C are sentence symbols.)

a. $A \vee B \to C \vDash A \to C$.

Answer: This is true, use a (reduced) truth table.

b. $A \to C \vDash A \vee B \to C$.

Answer: This is false. $v(A) = F$, $v(B) = T$, $v(C) = F$ is the only counterexample.

c. $(A \to B) \wedge (B \to C) \vDash \dashv (C \to B) \wedge (B \to A)$.

Answer: This is false. $v(A) = T$, $v(B) = F$, $v(C) = F$ is one of the four counterexamples.

d. $\vDash ((A \to B) \to A) \leftrightarrow A$.

Answer: This is true, use a (reduced) truth table.

2. True or False?

____ **a.** If α and β are both tautologies, then $\alpha \vDash \dashv \beta$. *Answer: True*

____ **b.** If Σ_1 and Σ_2 are both satisfiable, then $\Sigma_1 \vDash \dashv \Sigma_2$. *Answer: False*

____ **c.** If $\{\alpha, \beta\} \vDash \gamma$, then $\alpha \vee \beta \vDash \gamma$. *Answer: False*

____ **d.** If $\alpha \vee \beta \vDash \gamma$, then $\{\alpha, \beta\} \vDash \gamma$. *Answer: True*

____ **e.** If $\{\alpha, \beta\} \vDash \gamma$, then $\alpha \vDash \beta \to \gamma$. *Answer: True*

____ **f.** If $\alpha \vDash \beta \to \gamma$, then $\{\alpha, \beta\} \vDash \gamma$. *Answer: True*