Math 20F - Study Outline: Basic Skills

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Now updated with material through the final exam!

This is a list of the basic "skills" you should master for Math 20F. I have tried to make the list complete, but of course you are also responsible for items that were inadvertantly omitted. There is a list of lecture topics on the course web page that you may also use for review purposes.

In addition to these skills, You are expected to know definitions and theorems and how to apply the definitions and theorems appropriately. You are responsible for material from the textbook, material in the two course handouts, and the material covered in class.

Chapter 1. :

- Convert a system of linear equations to matrix form, and vice-versa.
- Convert a matrix to reduced for echelon form or to RREF.
- Solve an (R)REF system by back substitution.
- Determine the number of solutions to a system of equations.
- Perform row operations.
- Perform matrix operations (addition, multiplication, scalar multiplication, etc.)
- Compute A^T.
- Determine if a matrix is singular.
- Compute A^{-1} if it exists.
- Work with elementary matrices and know their correspondence to elementary row operations.
- Put a matrix in LU form (if it has an LU form)
- Work with partitioned (i.e., blocked) matrices.

Chapter 2. :

- Calculate a determinant using cofactors.
- Calculate the determinant of a matrix using row operations.
- Calculate the determinant of a 2×2 matrix.
- Know the effect of row and column operations on the determinant.
- (We skipped Cramar's rule, and you are not responsible for knowing it.)

Chapter 3. :

- Determine if a subset of a vector space is a subspace. Know the closure conditions for a subspace.
- Know how to use vector space properties. (You do not need to memorise the list of axioms for a vector space.)
- Find the null space of a matrix. Determine its dimension, i.e. the nullity of the matrix.
- Determine if a given set of vectors is a spanning set for $\mathbb{R}^n$.

End of midterm #1 material

- Determine if a given set of vectors are linearly independent.

- Determine if a given set of vectors is a basis for $\mathbb{R}^n$.
- Given a set of vectors, find a linearly independent subset.
- Work with the vector spaces $R^{m\times n}$, P_n, $C[a,b]$, $C^n[a,b]$.
- (We skipped the Wronksian, and you are not responsible for knowing it.)
- Determine the dimension of a subspace.
- Find a basis for a subspace.
- Perform a change of basis.
- Find the matrix that performs a change of basis.
- Calculate the rank of a matrix.
- Find the row space, column space, null space and $N(A)$ of a matrix. Determine the dimensions of these spaces.

Chapter 4. :

- Represent a linear operator by a matrix.
- Find the matrix representation of a rotation.
- Express dot product and cross product with a matrix representation.
- Determine if a given transformation is linear.
- Find the image and kernel of a transformation. (Kernel is the same as nullspace and image is the same as range. This will not be on the midterm.)
- (We skipped homogeneous coordinates and you do not need to know them.)
- (We have skipped, at least for now, similarity in section 5.3.)

Chapter 5. :

- Compute scalar products.
- Find the magnitude of a vector.
- Find the angle between two vectors.
- Find the scalar and vector projection of a vector onto another vector.
- Find the orthogonal complement of a subspace.
- Know the complementary properties of $R(A^T)$ and $N(A)$, and of $R(A)$ and $N(A^T)$.
- Solve least squares problems.
- Find the best linear fit to data.
- Find the best quadratic fit to data.
- Find the projection of a vector $\mathbf{b}$ onto a subspace given as the span of arbitrary vectors.
- Find the projection of a vector $\mathbf{b}$ onto a subspace given as the span of orthogonal vectors. (Also, onto a subspace given as the span of orthonormal vectors.)
- Recognize and use inner product notation.
- (For now at least, we skipped the use of function spaces as inner product spaces in section 5.4.)
- Determine if a set of vectors is orthogonal.
- Determine if a set of vectors is orthonormal.

..... End of midterm #2 material .....

- Determine if a matrix is orthogonal.
- Use the matrix method to find the projection of $\mathbf{b}$ onto a subspace given as a span of orthogonal vectors.

- (We skip, for now at least, permutation matrices and orthogonality in vector spaces of functions on page 275 and pages 279-285.)
- Solve least squares problems with orthogonal matrix.
- Gram-Schmidt method or modified Gram-Schmidt method to find an orthonormal basis of a subspace.
- Find the eigenvalues of a matrix.
- Find the eigenvectors or eigenspace corresponding to an eigenvector.
- Compute the trace of matrix.
- Compute the product and sum of the eigenvalues of a matrix.
- Find complex eigenvalues.
- Determine if a matrix is diagonalizable. If so, find a diagonalization.
- Use similarity to convert matrices to different representation.