Name: Tuesday section time:
Student ID:

Math 20F - Linear Algebra - Winter 2003

Quiz #$6\frac{6}{10}$ — March 11

Do not hand in this quiz: it is for self-assessment.

Try this quiz without referring to the answers (on back of paper copy) first!

1. Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

   Find all the eigenvariables of A, and an associated eigenvector for each eigenvector. Check your answers by computing $A\mathbf{x}$ for each eigenvector $\mathbf{x}$.

   ANSWER:

   The characteristic polynomial of A is $\det(A-\lambda I) = (1-\lambda)(2-\lambda)(4-\lambda)$. *Thus, the eigenvariables are* $\lambda_1 = 1$, *and* $\lambda_2 = 2$, *and* $\lambda_3 = 4$. *The associated eigenvectors are:* $\mathbf{x}_1 = (1,0,0)^T$, *and* $\mathbf{x}_2 = (2,1,0)^T$, *and* $\mathbf{x}_3 = (1,0,3)^T$. *(Your answers for the eigenvectors may differ by being multiplied by any non-zero scalar.)*

2. Repeat the above problem with $A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$.

   ANSWER:

   $\det(A-\lambda I) = (1-\lambda)^2 - 9 = \lambda^2 - 2\lambda - 8$. *The eigenvalues are the two roots of this characteristic polynomial, which are* $\lambda_1 = 4$ *and* $\lambda_2 = -2$. *The associated eigenvectors are:* $\mathbf{x}_1 = (1,1)^T$ *and* $\mathbf{x}_2 = (1,-1)^T$. *(Again, your answers for the eigenvectors may differ by being multiplied by any non-zero scalar.)*