Math 20F - Linear Algebra - Winter 2003

Quiz $\#6\frac{1}{2}$ Answers — March 4

1. Consider the following table of data values.

x	-1	0	1	2
y	0	4	2	4

Find the best linear least squares fit to the data. That is, find the linear function $f(x) = c_0 + c_1 x$ that best fits the data in the least squares sense.

ANSWER: Let $A = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$ *and* $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \\ 2 \\ 4 \end{pmatrix}$. *We need to solve* $A\binom{c_0}{c_1} = \mathbf{b}$. *To do this, we solve* $A^T A\mathbf{x} = A^T\mathbf{b}$. *Now,* $A^T A = \begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix}$, *and* $A^T\mathbf{b} = \binom{10}{10}$, *so we use row operations:*

$$\left(\begin{array}{cc|c} 4 & 2 & 10 \\ 2 & 6 & 10 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & 1 & 5 \\ 1 & 3 & 5 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 1 & 5 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -5 & -5 \end{array}\right)$$

Solving by backsubstitution gives $c_1 = 1$ *and* $c_0 = 2$. *So the answer is:* $f(x) = 2 + x$.

2. Let $\mathbf{u}_1 = (1,1,1)^T$ and $\mathbf{u}_2 = (1,-1,0)^T$. Are these vectors orthogonal? Orthonormal? Let $\mathbf{x} = (0,1,1)^T$. Find the projection $\mathbf{p}$ of $\mathbf{x}$ onto the subspace $Span(u_1, u_2)$.

ANSWER: $\mathbf{u}_1$ *and* $\mathbf{u}_2$ *are not unit vectors, so we normalize them by dividing by their magnitudes to get* $\mathbf{v}_1 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^T$ *and* $\mathbf{v}_2 = (1/\sqrt{2}, -1/\sqrt{2}, 0)^T$. *Then,* $\mathbf{v}_1 \cdot \mathbf{x} = 2/\sqrt{3}$ *and* $\mathbf{v}_2 \cdot \mathbf{x} = -1/\sqrt{2}$. *Hence,*

$$\mathbf{p} = \frac{2}{\sqrt{3}}\mathbf{v}_1 + \frac{-1}{\sqrt{2}}\mathbf{v}_2 = (2/3, 2/3, 2/3)^T + (-1/2, +1/2, 0)^T = (1/6, 7/6, 2/3)^T.$$