Name: Tuesday section time:
Student ID:

Math 20F - Linear Algebra - Winter 2003

Quiz #5 — February 18

(Do not discuss the quiz with students who haven't taken it yet – until 8:00pm.)

(Do not calculate more than is necessary to answer a problem! For one of these problems the answer(s) should be obvious from inspection.)

1. Let $\mathbf{v}_1 = (1,\ -1)^T$ and $\mathbf{v}_2 = (1,\ 2)^T$, so $\mathbf{v}_1$, $\mathbf{v}_2$ are a basis for $\mathbb{R}^2$.
   Let $\mathbf{x} = (3,\ 2)^T$.
   What are the coordinates $\mathbf{x}$ with respect to the basis $\mathbf{v}_1$, $\mathbf{v}_2$?

   METHOD: You need to find a_1, a_2 *so that* $\mathbf{x}_1 = a_1\mathbf{v}_1 + a_2\mathbf{v}_2$. *For this you solve the matrix equation* $\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. *This can be done either by row operations, or by inverting the* 2×2 *matrix.*

   ANSWER: The coordinates of $\mathbf{x}$ *w.r.t. the basis* $\mathbf{v}_1, \mathbf{v}_2$ *are* $(\frac{4}{3}, \frac{5}{3})$. *That is,* $\mathbf{x} = \frac{4}{3}\mathbf{v}_1 + \frac{5}{3}\mathbf{v}_2$.

2. Let A be the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \\ 3 & 3 \end{pmatrix}$.

   a. What is the dimension of the row space of A?
   b. What is the dimension of the column space of A?
   c. What is the rank of A?

ANSWER: All three of these questions have the same answer: 2.

To see this, note that the dimension of the column space is clearly equal to two, since the two columns are linearly independent.

Another way to see this is to note that the row space is a subset of $\mathbb{R}^2$ *and hence has dimension at most two. Clearly, however, it does not have dimension one, hence it must have dimension two.*